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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Edited by

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Mount Wilson Observatory of the Carnegie Institution of Washington

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Yerkes Observatory of the University of Chicago

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MAY 1931

A STUDY OF THE [illegible] EFFECT IN ECLIPSING BINARIES

AN INVESTIGATION OF [illegible] SPECTRA IN SUNSPOTS

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THE ASTROPHYSICAL JOURNAL

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A STUDY OF THE REFLECTION EFFECT IN ECLIPSING BINARIES

By EUGENE W. PIKE

ABSTRACT

By means of *approximations*, the equations of *Milne* for *re-radiation from stellar surfaces* are applied to *close binaries*.

The *variation* of the *reflection effect* with *phase* is computed, and a *more accurate approximation* than the one in *present use* is proposed.

The *exaggerated darkening* at the *limb* shown by the *secondary disk*, as seen at *secondary minimum*, is illustrated, and its *possible effect* on *solutions* of the *light-curve* discussed.

An *approximate correction* to the *spectroscopically determined masses* of a binary, to account for the *asymmetrical lighting* of the *secondary* as seen at *quadrature*, is computed.

A *large discrepancy* between the *computed* and *observed absolute magnitudes* of the *reflection effect* is *illustrated* and *discussed*.

Eclipsing binaries of the "giant and dwarf" type, that is, close binaries where the secondary star is as large or larger than the primary, but of far lower surface brightness, show generally a slight increase in total light as the secondary star passes from inferior to superior conjunction. This phenomenon, known as the "reflection effect," was first noted by R. S. Dugan[1] in 1908, during his study of RT Persei, and was correctly explained by him as due to the illumination of the inner side of the secondary star by the primary. J. Stebbins[2] and C. Nordmann[3] also discovered and explained this effect quite independently.

[1] *Science*, **28**, 854, 1908; *Princeton University Observatory Contribution*, No. 1, 1910.

[2] *Astrophysical Journal*, **32**, 213, 1910.

[3] *Bulletin astronomique*, **27**, 145, 1910.

The mathematical calculation of the magnitude of this effect and its variation with phase has been undertaken by Stebbins,[1] A. S. Eddington,[2] and E. A. Milne.[3] Stebbins did his work about 1912, and considered that the major part of the effect was ascribable to diffuse reflection from the photosphere of the secondary, the increased temperature of the secondary surface being negligible. On this basis he carried through a rigorous solution of great complexity, using various laws of reflection derived from planetary photometry. Eddington considered the physical aspects of the problem more completely, but limited his calculations to the case of widely separated stars. He argued that the heat-albedo of the secondary must be unity, since its internal temperatures and gradients are too great to be disturbed by the small amount of energy incident from the primary. He also made the important point that the visual efficiency of the re-radiated light might be greatly different from that of the light from the primary.

Milne, applying the equations of radiative equilibrium, shows that solutions of these equations are superposable in an infinite homogeneous star, and makes a closely approximate solution of these equations for the case of a parallel beam of radiation incident on a stellar surface, deriving the following formula for the variation in intensity of the re-radiated light with angle:

$$L(\alpha, \varphi)=\frac{E'}{\pi}\cdot\frac{(\cos\alpha+\frac{1}{2})(\cos\varphi+\frac{1}{2})}{\cos\alpha+\cos\varphi}\cos\varphi\,, \tag{1}$$

where α is the angle between the incident beam of radiation and the normal, φ is the angle between the normal and the cone of re-radiated energy under consideration, E' is the energy incident on unit area of the stellar surface, and $Ld\omega$ is the energy re-radiated into a cone of solid angle $d\omega$ by a unit area of the surface.

The values of L computed from equation (1), in terms of E'/π as unity, are given in Table I for various values of φ and α. For comparison, the values of L computed from Lambert's law,

$$L(\varphi)=\frac{E'}{\pi}\cos\varphi\,, \tag{2}$$

[1] *Probleme der Astronomie* (Festschrift für H. von Seeliger), p. 442, J. Springer, 1924.

[2] *Monthly Notices of the Royal Astronomical Society*, **86**, 320, 1926.

[3] *Ibid.*, **87**, 43, 1927.

and from the formulae

$$L(\varphi)=\frac{3E'}{4\pi}\left(1+\frac{\cos\varphi}{2}\right)\cos\varphi\,, \tag{3}$$

and

$$L(\varphi)=\frac{6E'}{5\pi}\left(1-\frac{\cos\varphi}{4}\right)\cos\varphi\,, \tag{4}$$

which give one-third darkening and one-third brightening, respectively, at the limb of a uniformly illuminated sphere, are set out.

Lambert's law is obviously a fair approximation to the accurate equation, especially where considerable ranges of α and φ are con-

TABLE I

φ	0°	30°	45°	60°	75°	90°
$L(2)$	1.00	0.87	0.71	0.50	0.26	0
$L(3)$	1.13	.93	.72	.47	.22	0
$L(4)$	0.90	.81	.70	.52	.29	0
$L(1)$; $\alpha=0°$	1.13	.96	.75	.50	.23	0
$\alpha=30$	1.10	.94	.74	.50	.24	0
$\alpha=45$	1.06	.91	.73	.50	.25	0
$\alpha=60$	1.00	.87	.71	.50	.26	0
$\alpha=75$	0.91	.80	.67	.50	.29	0
$\alpha=90$	0.75	0.69	0.61	0.50	0.38	0

sidered. Used in computing the variation with phase of the reflection effect, it will give values too low at secondary minimum, and too high at quadrature. Since Milne's equation leads to very complicated integrals except in simple cases, Lambert's law will be used in this discussion.

Consider a "giant and dwarf" binary, as shown in Figure 1. Let O_b[1] and O_f be the centers of the stars, P_b and P_f mutually visible points on their surfaces. If E_b is the total energy radiated from unit area of the primary (darkening at the limb of the primary is neglected), then the energy incident on unit area of the secondary at P_f from an area ds of the primary at P_b is

$$dE_r=\frac{E_b}{\pi}\cdot\frac{\cos i\cos j}{x^2}\,ds\,, \tag{5}$$

[1] In all the formulae to follow these subscripts are used: b refers to the primary star, f to the secondary or the light from within it, r to the reflected radiation, and s to the illuminated face of the secondary.

where x is the distance P_bP_f, i and j are the angles between P_bP_f and the normals at P_f and P_b, respectively. Integrating this over the portion of the primary visible from P_f, the following expression results:

$$E_r(\beta)=E_b\left(\frac{r_b}{z}\right)^2\left(\frac{a^2-r_f^2-z^2}{2r_fz}\right), \tag{6}$$

where $\beta=\widehat{P_fO_fO_b}$, $a=\overline{O_fO_b}$, the r's are the respective radii, and

$$z^2=\overline{O_bP_f}^2=a^2+r_f^2-2ar_f\cos\beta .$$

This formula (6) is valid while P_f is within the common inner tangent cone to the two stars. Outside this cone E_r must be evaluated in elliptic functions, a refinement not worth the labor, since E_r is small and dropping rapidly in this region.

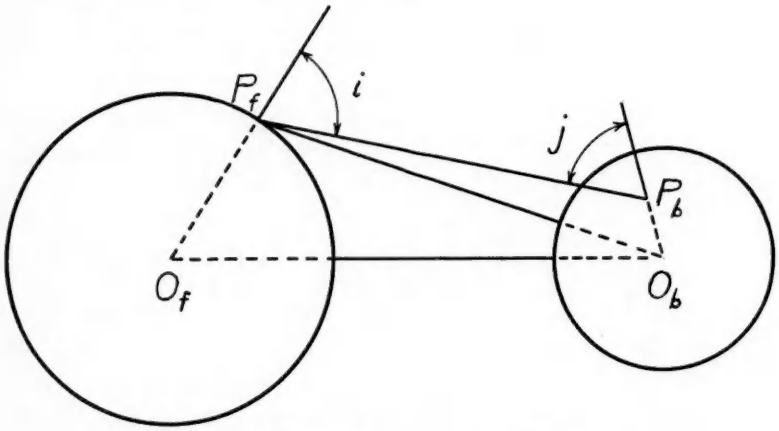

FIG. 1.—Illumination of secondary by primary star

Equation (6), with its discontinuity, would lead to very complicated mathematical forms should one attempt to use it in further calculations. It can be shown, however, that in the great majority of cases the formula

$$E_r(\beta)=A_bE_b\cos^n\beta , \tag{7}$$

where A_b and n are adjustable constants, is an entirely adequate first approximation. The value of n in this formula is obviously a func-

tion of r_f/a alone. For convenience in computation a few values are given in Table II.

Formula (7) may be used to compute the variation of the reflection effect with phase. Taking as variables the latitude and longitude angles about the pole of the plane containing the line of centers and the line of sight, the integration is straightforward. As a numerical check, formula (1) may be integrated directly at secondary

TABLE II

r_f/a	0.20	0.24	0.27	0.30	0.33	0.36
n	2.1	2.4	2.7	3.0	3.4	3.8

minimum, with the simplifying approximation $\alpha=\beta$, and yields a result only 7 per cent higher, for values of n between 1 and 4. As will be seen later, this discrepancy is negligible.

TABLE III

θ	0°	60°	90°	120°	150°	180°	Abs
M_r	0	0.11	0.32	0.60	0.89	1.00	.....
L_r	.....	.....	.....	.....	.....	.....	.....
$(n=1)$	0	.11	.32	.60	.88	1.00	0.67
$(n=2)$	0	.07	.25	.56	.87	1.00	.50
$(n=3)$	0	.04	.22	.55	.87	1.00	.40
$(n=4)$	0	.03	.18	.52	.86	1.00	0.33
L_r'	0	.25	.50	.75	.93	1.00	.....
L_r''	0	0.04	0.21	0.53	0.87	1.00	.....

In Table III the values of L_r given by this integration are set out as a function of n and the phase angle θ (measured from primary minimum), the values at secondary minimum being set equal to unity. For comparison

$$M_r=\frac{(\sin\theta-\theta\cos\theta)}{\pi}, \tag{8}$$

given by Milne as the result of direct integration of equation (1) for $n=1$,

$$L_r'=\cos^2\frac{\theta}{2}, \tag{9}$$

the present approximation, and

$$L_r'' = \cos^{\frac{9}{2}} \frac{\theta}{2}, \tag{10}$$

proposed as a better approximation, are tabulated. In the extreme right-hand column the absolute values of L_r at secondary minimum in terms of $A_b L_b r_f^2 / r_b^2$ as unity are given.

Thus far the computations have all been in terms of total energy. With ordinary means of observation, the efficiency of the reflected energy will vary with the surface temperature over the face of the secondary. In general, however, equation (7), with a slight change in n, is still an adequate approximation. Since the solutions of the equations of radiative equilibrium are additive,

$$T_s^4 = T_f^4 + T_r^4, \tag{11}$$

where $\sigma T_r^4 = E_r$. The empirical formulae

$$M_v = \frac{29{,}500}{T} - 5 \log R - 0.08, \tag{12}$$

$$M_p = \frac{36{,}700}{T} - 5 \log R - 0.72, \tag{13}$$

derived by H. N. Russell,[1] give the absolute magnitude in terms of the temperature, and can be used to compute efficiencies.

As an example of the methods outlined above, consider the case of RS Vulpeculae. The constants from the best present solution are:[2] $r_b = 0.20\ a$, $r_f = 0.27\ a$, J_b/J_f (vis.) $= 11.6$, primary spectrum B8, inclination 79°, $m_b = 4.59 \odot$, $m_f = 1.44 \odot$. Russell's[3] temperature scale gives $T_b = 12{,}500°$. From formula (12), $T_f = 5900°$, J_b/J_f (bolometric) $= 20$. Formula (6) gives E_r, formula (11) the effective temperature, and formula (12) the visual efficiencies. This visual efficiency is applied to E_s ($E_s = E_f + E_r$), and the visual reflected magnitude $V_r = V_s - V_f$. These various quantities are shown in Table IV.

[1] Russell-Dugan-Stewart, *Astronomy* (Ginn & Co., 1927), II, 732.

[2] Dugan, *Princeton University Observatory Contribution*, No. 6, 1924.

[3] Russell-Dugan-Stewart, *op. cit.*, p. 753.

Interpolation in Table II gives

$$L_r(\text{vis.}) = 0.44 \times 0.136 \times \left(\frac{0.27}{0.20}\right)^2 \times L_b ,$$

$$= 0.11\ L_b .$$

TABLE IV

β	F_r	T_s	V_s	V_r
0°.....	$0.075E_b$	7400°	$0.222V_b$	$0.136V_b$
15.....	.068	7300	.210	.124
30.....	.048	7000	.180	.094
45.....	.028	6600	.144	.058
90.....	0.00	5900	0.086	0.00

$E_r = 0.075E_b \cos^{2.7}\beta$ $\qquad$ $V_r = 0.136V_b \cos^{2.6}\beta$

$E_f = 0.05E_b$ $\qquad$ $V_f = 0.086V_b$

Because of the inclination of the orbit, the maximum theoretically observable value of L_r is $0.10L_b$, in complete agreement with the observed value $0.10L_b$. This agreement is probably fortuitous, for the probable errors in both values are large.

TABLE V

Star	L_r		Star	L_r	
	Comp.	Obs.		Comp.	Obs.
RS CVn*.....	$0.028L$	$0.054L$	β Per||.....	$0.11L$	$0.03L$
TV Cas†.....	.12	.09	X Tri¶.....	.10	.02
RZ Cas‡.....	.08	.06	T LMi†.....	.07	.00
U Cep§.....	0.10	0.06	RY Aqr**.....	0.07	0.00

* B. F. Sitterly, *Contributions of the Princeton University Observatory*, **11**, 21, 1930.
† R. J. McDiarmid, *ibid.*, No. 7, 1924.
‡ R. S. Dugan, *ibid.*, No. 4, 1916.
§ *Ibid.*, No. 5, 1920.
|| D. B. McLaughlin, *Astrophysical Journal*, **60**, 28, 1924.
¶ R. S. Dugan, *op. cit.*, No. 8, 1927.
** *Ibid.*, No. 6, 1924.

Other eclipsing binaries, equally well observed, show large discrepancies. These are shown in Table V. Eddington[1] noted similar discrepancies in a number of other stars, as the result of approximate calculations, and ascribed them tentatively to re-radiation of the high-temperature energy from the primary with unchanged distri-

[1] *Loc. cit.*

bution of energy. This explanation is inadequate to account for the large ratios shown in Table V, requiring a surface temperature of the primary in excess of 25,000°, while the spectral types are A or late B. In addition, it is difficult to conceive a mechanism for this process on present theories of the photosphere.

The next step would seem to be an examination of Milne's postulates. His proof of the additivity of solutions of the equations of radiative equilibrium is based on an infinite, homogeneous star, and his solutions indicate a finite rise of temperature in the deep interior. It has been suggested that under these conditions there might be a sidewise flow of energy which, though infinitesimal compared to the total flux at that depth, might be a very considerable fraction of the flux outward through the surface. Carrying this farther, Professor H. N. Russell has suggested that the assymmetry forced upon the secondary star by prolonged heating of one side by the primary, in the generally accepted condition of an infinite synodic period for the secondary, might bring about an alteration in its fundamental internal equilibrium and constitution, so that the assumption of homogeneity would be violated. It is also possible that an assymmetry could persist from the event of fission. Any quantitative discussion of these hypotheses would be tremendously complicated.

Professor Russell also suggested that the secondary might be rotating with respect to the axis of the binary, in which case none of these considerations would be applicable. If the period of this rotation were long, the reflection effect might be expected to attain its full computed value, while increasing angular velocity would reduce the magnitude of the effect and introduce in addition a shift of phase with complete disappearance of both as a limit.

In the case of RS Vulpeculae and similar stars, formula (10), and other formulae to be developed, may be applied with some confidence. Stars like TV Cassiopeiae, where the agreement is within the experimental error, can be treated by adjustment of A_b. For stars showing greater discrepancies it is difficult to speak. In the absence of any theoretical information, the present methods of treatment are as good as any. Fortunately, the greater the discrepancy, the smaller the error introduced by rough approximations. In general, eclipsing binaries of the "giant and dwarf" type have a computed reflection

effect of $0.05L$ or greater. If the star under consideration shows a reflection effect as large or larger than this minimum, formula (10) should be used. If it is less, formula (9) is adequate.

Eddington[1] also pointed out that the spectroscopic values for the masses of eclipsing binaries were probably in error, since the "center of gravity" of the light from the secondary as seen at quadrature is displaced from the center of the disk by the reflection effect. On the assumption that the point whose velocity is indicated by the absorption lines is the "center of gravity" of the light from the secondary, this error can be computed.

Since this computation deals with only one spectrum line at a time, the relative efficiencies of the various emitting surfaces may be derived directly from Planck's Law. These efficiencies are given by the ratio

$$U_{\lambda, T}=\frac{1/\lambda^5}{T^4(e^{hc/\lambda kT}-1)} . \tag{14}$$

Using this in place of equation (12), one can derive

$$S_r=A_b'S_b \cos^m \beta . \tag{15}$$

The insertion of this, in place of equation (7), into Table III results in $(L_r)_S(m, \theta)$. By simple algebra the displacement $G_r r_f$ of the "center of gravity" of the reflected light at quadrature is given,

$$G_r r_f=\frac{r_f(L_r)_S(m+1,90^\circ)}{(L_r)_S(m, 90^\circ)} . \tag{16}$$

The displacement factor for the light of the whole secondary star is

$$G_f=\frac{(L_r)_S(m+1,90^\circ)}{(L_r)_S(m,90^\circ)+L_f} . \tag{17}$$

Milne[2] proves that the increase in surface temperature of the secondary due to the impinging radiation is given by

$$T_s'^4=T_f'^4(1+2E_r/E_f) , \tag{18}$$

[1] *Ibid.*

[2] *Ibid.*, **87**, 43, 1927.

in contradistinction to equation (11), which is true for the effective temperature. Eddington[1] had already suggested that the high-temperature radiation from the primary would induce high-temperature lines in the secondary spectrum. In many cases (see RS Vulpeculae)[2] spectroscopic observers report that the spectrum of the secondary is very like that of the primary. In the extreme case of this, where the line being measured is absorbed over the illuminated hemisphere of the secondary only, the displacement factor becomes

$$G_s = \frac{(L_r)_S(m+1,90^\circ)+L_f/\pi}{(L_r)_S(m,90^\circ)+L_f/2} . \tag{19}$$

For any given line the displacement will lie between these two values, and must be determined with consideration for its nature and its displacement relative to other lines. It should also be noted

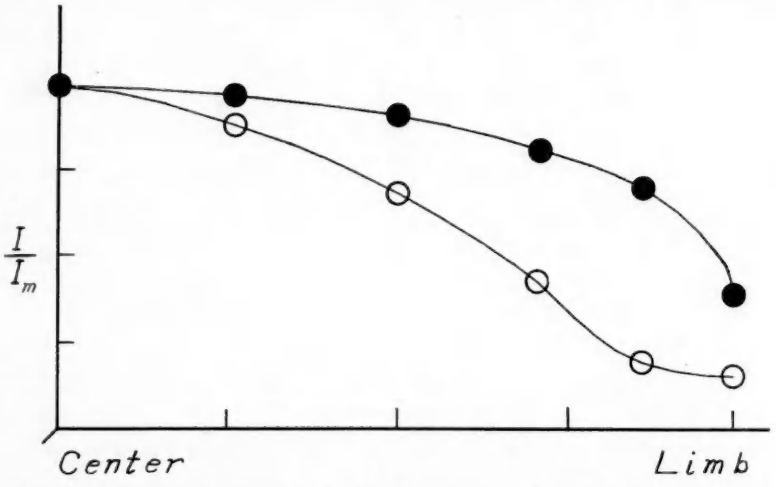

FIG. 2.—Darkening toward the limb. Filled circles, six-tenths darkening; open circles, "reflection" darkening.

that these formulae are correct only at quadrature, so that the observed velocities of the secondary will not necessarily lie on a sinusoidal curve, even though the orbit be circular.

In the case of RS Vulpeculae, on the assumption of a mean value

[1] *Loc. cit.*

[2] Dugan, *Princeton University Observatory Contribution*, No. 6, 1924.

for λ of 4500 A.U., the maximum and minimum values for the masses are $m_b = 7.77\odot$, $m_f = 1.99\odot$, and $m_b = 5.09\odot$, $m_f = 1.54\odot$, respectively. The directly derived masses are $m_b = 4.59\odot$, $m_f = 1.44\odot$. Dr. J. S. Plaskett, who measured the lines, states that the secondary spectrum was very similar to the primary, so that the true masses are probably notably greater than the minimum given above.

Still another consequence of the reflection effect is an exaggerated darkening at the limb of the secondary disk as seen at secondary minimum. If, as is often the case, $A_b E_b \gtrsim E_f$, this darkening may introduce large errors in J_b/J_f as determined from the relative depths of the minima. Particularly will this be the case if the eclipse is grazing annular or shallow partial, so that the average brightness of the eclipsed portion is greatly different from the average brightness of the whole disk. In Figure 2 the intensity for the typical case $A_b E_b = 3E_f$, $n = 3$ is plotted against the radial distance sin β. For comparison the six-tenths darkening observed in the sun is also plotted.

In conclusion, it is a pleasure to acknowledge the sustaining interest of Professor R. S. Dugan, and the helpful criticism of all the Observatory staff.

PRINCETON UNIVERSITY OBSERVATORY
PRINCETON, N.J.
February 4, 1931

AN INVESTIGATION OF MOLECULAR SPECTRA IN SUN-SPOTS[1]

By R. S. RICHARDSON

ABSTRACT

Identification of bands known to exist in the spectra of sun-spots and the reversing layer.—About 55 per cent of the lines in the λ 5165 Swan band (C_2), λ 5167, λ 4954 bands of *TiO*, λ 5211 *MgH* band, and the A band of *CaH* were identified with lines in the solar spectrum with an average difference of 0.014 A. These data were the basis of a search for new bands in the spectra of sun-spots and the reversing layer.

Evidence for the presence of SiF, AlO, AlH, ZrO, and H_2 in sun-spots and the reversing layer.—Identification of 28 (82 per cent) of the stronger unobscured lines in the (0, 0) λ 4368.22 band of *SiF* with previously unidentified solar lines made probable the presence of fluorine in the sun. The more prominent features of the λ 4368.22 band are completely obliterated by strong atomic lines, but the region near the head of another strong *SiF* band at λ 4400 is unobscured, and here the correspondence in position and intensity between the solar and *SiF* lines is very marked.

Out of 291 unobscured lines in the λ 4842, λ 4648, and λ 5079 bands of *AlO*, 141, or 48 per cent, were identified in the spot spectrum with an average difference of 0.016 A. Since this is much higher than the number of chance coincidences in this region, it is concluded that *AlO* is present in sun-spots.

AlH is probably also present in sun-spots and the reversing layer, since the percentage of lines identified is much higher than would be expected by chance and the laboratory and solar intensities are in close agreement.

The *ZrO* band at λ 6474 is the strongest in stars of class S and the most favorably placed for identification in the sun-spot spectrum. The number and nature of the coincidences between arc lines near the head of this band and faint spot lines furnish favorable evidence for the presence of *ZrO* in sun-spots.

The H_2 lines of intensities 7, 8, 9, 10 in the list of Gale, Monk, and Lee were compared with spot lines in the region λ 4500 to λ 5500. The percentage of identified lines of intensity 7, 8, and 9 is about the same as the number of chance coincidences; 86 per cent of the lines of intensity 10 were identified, however, indicating that some of these may be real.

Probability of MgH and TiO in the reversing layer.—Fifty-two per cent of the *MgH* lines identified in the spot spectrum were also found in the disk spectrum. As this is considerably higher than the percentage of chance coincidences, *MgH* probably exists in the reversing layer.

The percentage of *TiO* lines found in the disk spectrum is about the same as that expected by chance. *TiO* is, therefore, probably confined entirely to sun-spots.

The presence in the spot spectrum of both the Swan bands of carbon and the TiO bands is noted. This is the only known example of the simultaneous occurrence of these bands in celestial spectra.

Temperatures of sun-spots and the reversing layer determined from the distribution of intensity in the λ 5165 Swan band.—A comparison of the observed and theoretical intensities of lines in the λ 5165 Swan band yielded temperatures of $4900° \pm 600°$ K for the sun-spot umbra and $6000° \pm 700°$ K for the reversing layer. From the position of maximum intensity in the λ 5165 Swan band in the spot and the reversing layer, the temperatures of the sun-spot umbra and the reversing layer were found to be $4900° \pm 750°$ and $5700° \pm 1600°$ K, respectively. The Rowland intensities of the band lines were calibrated by the method of Russell, Adams, and Moore and the temperature calculated from the slope of the log $(I/\bar{J})$ curve. This method gives temperatures of $4500° \pm 400°$ K for the sun-spot umbra and $5300° \pm 400°$ K for the reversing layer.

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 422.

1. INTRODUCTION

During the last thirty years the behavior of atomic lines in the spot and disk spectra of the sun has been intensively studied, but relatively little attention has been devoted to the numerous molecular lines in the solar spectrum. The *Revised Rowland Table*[1] gives the intensities in both spot and disk of atomic lines only. Certain molecular lines such as *CN*, *CH*, and *NH* are identified in the disk spectrum; but no attempt has been made to identify band lines that are either very faint in the disk spectrum or else entirely lacking, as, for example, *TiO*, *MgH*, and *CaH*. Further, there is nothing in the *Revised Rowland Table* to indicate whether the numerous faint unidentified lines are of atomic or molecular origin.

This paper contains the results of an investigation of molecular spectra in sun-spots, especially in the region λ 4500 to λ 5500. It was undertaken with the following objects in mind: (*a*) to identify lines in the λ 5165 Swan band of carbon (C_2), the λ 5211 band of *MgH*, the A band of *CaH*, and the λ 5167, λ 4954 bands of *TiO*, in order to ascertain their relative intensities and frequency of occurrence in the spot and disk spectra; (*b*) to use the data obtained from (*a*) in a search for new bands in the solar spectrum; (*c*) to determine the temperatures of sun-spots and the reversing layer from the distribution of intensity in the λ 5165 Swan band.

2. OBSERVATIONAL MATERIAL AND REDUCTION

The measurements of wave-length and estimates of intensity were made from a series of spot-spectrum plates taken with a nicol prism and quarter-wave plate in the third order of the 75-foot spectrograph at Mount Wilson. The linear scale is 1 A = 4.8 mm. The data for these plates are as follows:

Region	Date	Position of Spot	Gr. No.
λ 4899–4991	1917 February 7	30E, 14S	7977
4989–5103	1917 February 7	30E, 14S	7977
5082–5200	1919 February 13	6E, 9N	8833
5193–5302	1919 February 13	6E, 9N	8833
5288–5407	1919 February 13	6E, 9N	8833

[1] *Publications of the Carnegie Institution of Washington*, No. 396; *Papers of the Mount Wilson Observatory*, 3, 1928.

The positions of the spot lines were determined from the wave-lengths of the atomic lines as given in the *Revised Rowland Table*. These wave-lengths were then combined with some unpublished measurements of the spot spectrum from λ 4500 to λ 5500 taken with the Snow telescope at Mount Wilson. The linear scale for these plates is about 1 A = 1.5 mm. Since the unpublished wave-lengths had been referred to the Rowland scale, they were reduced to the international system before the two series of measurements were compared. Since the agreement in wave-length and intensity is very close for most of the lines, there was no uncertainty in identification.

The intensities used for the lines are the writer's eye-estimates based on the Rowland intensity scale. The adopted values are the means of independent estimates made when measuring in both the direct and the reverse positions. A line of intensity −3 (0000 as formerly written) is the faintest that can be seen. Since the molecular lines in the region studied are all included within the range −3 to 1, it seemed that some gain in accuracy could be made by estimating the intensities of these lines to a tenth of a Rowland unit. The ordinary classification appeared too rough to express the fine gradations of intensity of the faint band lines.

The plates show no appreciable change of sensitivity with wave-length over the short interval measured, and it is believed that any intensity differences arising from this source are within the errors of observation. A more serious error might arise from the fact that the plates covering the interval λ 5082 to λ 5407 were taken two years after those for the region λ 4899 to λ 5103 and on a different sun-spot. The two series overlap and have 129 lines in common. Comparison of the intensities of these lines in the two spots gives the following results:

Average intensity difference without regard to sign	0.24
Average intensity difference with regard to sign	−0.01
Number with positive difference	49
Number with negative difference	59
Number with zero difference	21

These differences are well within the errors of observation, and there is no evidence of a systematic difference. Consequently, no reduction has been made from one series to the other.

The nicol prism and compound quarter-wave plate clearly reveal the Zeeman effect by the staggered appearance of the atomic lines in the spot spectrum. For 11 per cent of the lines the presence or absence of the Zeeman effect is in doubt, usually because the line is a close blend. Since the components of the faint atomic lines in the spot spectrum have, in general, a greater separation than the stronger lines, even a weak atomic line could readily be distinguished from a host of neighboring band lines. For the strong lines (intensity 1 or greater), the presence or absence of the Zeeman effect was never in doubt, owing to the better visibility of these lines. Since molecular lines are not appreciably widened by the magnetic fields in sunspots (maximum intensity about 4200 gauss), atomic and molecular lines could easily be differentiated by simple inspection.

3. COMPARISON WITH THE "REVISED ROWLAND TABLE"

The wave-lengths of the lines measured from λ 4900 to λ 5400 which did not show the Zeeman effect, or for which the Zeeman effect was in doubt, were compared with those in the corresponding region in the *Revised Rowland Table*. The summarized results are:

Total number of lines measured	1848
Number of measured lines in *Revised Rowland Table*	718, or 39 per cent
Average difference in wave-length	0.011 A

When the extreme difficulty encountered in measuring many of the spot lines is considered, the agreement in wave-length is probably about as close as can be expected. In general, it is believed that the wave-lengths may be relied upon to 0.010 A; but a line may easily be in error by 0.035 A, depending upon the character of the line, background intensity, and proximity to other lines.

4. IDENTIFICATION OF MOLECULAR LINES IN THE SUN-SPOT SPECTRUM

Certain bands which were definitely known to exist in the spot spectrum have been investigated, with the results included in Table I. The wave-lengths for the *TiO*[1], *MgH*[2], and C_2[3] bands were

[1] *Physical Review*, **33**, 701, 1929. [2] *Ibid.*, **29**, 413, 1927.

[3] *Philosophical Transactions of the Royal Society of London*, A, **226**, 157, 1927.

derived from measurements made within the last six years. The *CaH* wave-lengths are from E. Hulthén's[1] analysis of the A and B bands of *CaH* and from some older unpublished measurements by C. M. Olmsted. There still remain many unidentified lines which probably belong to the bands of these four molecules, but since they are slightly irregular in wave-length or intensity, they have been rejected. The percentage of *MgH* and *CaH* lines identified is higher because of the greater intensity of these bands in the spot spectrum.

The percentages and average differences in Table I may be used

TABLE I

SUMMARY FOR BANDS DEFINITELY KNOWN TO EXIST IN THE SOLAR SPECTRUM

BAND	MOLECULE	UNOBSCURED LINES	NUMBER IDENTIFIED	PERCENTAGE IDENTIFIED	AVERAGE DIFFERENCE	AVERAGE INT.	
						Disk	Spot
λ 4954.....	*TiO*	252	131	52	0.013 A	.....	−1.6
5167.....	*TiO*	295	127	43	.013	.....	−2.1
5165.....	C^2	400	176	44	.014	−2.2	−1.4
5211.....	*MgH*	146	105	72	.014	−2.6	−0.6
6903*.....	*CaH*	190	143	75	0.014	.....	+0.4
Totals and means....	.....	1283	682	57	0.014	−2.4	−1.1

* Only lines in the A band of *CaH* were identified.

as a guide in searching for new bands that may exist in the solar spectrum. Such bands must be very faint to have escaped previous detection; hence, many of the lines will not be seen at all or else will be obliterated by stronger lines. But near the band head, where the lines are generally most numerous and intense, we may expect to identify approximately 50 per cent of the lines with an average difference of about 0.015 A. With these data for guidance, an attempt has been made to find in the solar spectrum evidence for the existence of new bands. A special search was made for bands already known to occur in stellar spectra but not yet observed in the solar spectrum, and for bands whose presence in the solar spectrum would be of exceptional importance. Band lines in the interval λ 4500 to λ 5500 could be compared directly with the table of spot wave-lengths available

[1] *Physical Review*, **29**, 97, 1927.

for this region. Outside these limits, a search was made on the Mount Wilson spot-spectrum map at the positions of the stronger band heads. This map is an excellent enlargement of the spot plates taken in the second and third orders with the 75-foot spectrograph. The scale is 1 A = 1 cm. Evidence was found for the existence of *SiF*, *ZrO*, *AlO*, *AlH*, and possibly H_2 in sun-spots and in the reversing layer.

5. RESULTS OF A SEARCH FOR NEW BANDS IN THE SOLAR SPECTRUM

Silicon fluoride.—Atomic lines of the halogens have not been found in the solar spectrum, but since the molecular spectra of these elements are very extensive, it was thought that possibly one or more of them might be found in the spectra of sun-spots and the reversing layer. Search was made for the band spectra of halogens for which accurate wave-lengths and intensities are available. Definite evidence was found for one compound only—*SiF*. The probable presence of this molecule in the solar atmosphere and the absence of the others are presumably due to its high heat of dissociation (about 5 volts) and the abundance of silicon in the sun.

The laboratory wave-lengths and intensities of the *SiF* lines were taken from Johnson and Jenkins'[1] analysis of these bands. They give wave-lengths to three decimals for the lines of the strong (0,0) λ 4368.22 band of the α system of *SiF*. The wave-lengths of the lines in the other *SiF* bands are much less accurate and hence their detailed identification has not been attempted. No effort was made to identify lines in the band from λ 4377 to λ 4380 on account of the appearance of a faint component on the more refrangible side of the main lines in this region. This component probably influences the measurements of the wave-lengths and intensities of the main lines and makes them less accurate than the measurements of the lines which do not have this satellite. Only seven unobscured lines were rejected by excluding this portion of the band.

An examination of the unidentified lines of intensity 0 or less in the *Revised Rowland Table*, in the region over which the λ 4368.22 band of *SiF* extends, showed that about 90 per cent fall very close

[1] *Proceedings of the Royal Society of London*, A, **116**, 327, 1927.

to *SiF* lines. Conversely, a high percentage of the stronger *SiF* lines can be satisfactorily identified with previously unidentified solar lines. Tables II*a* and II*b* summarize the evidence for *SiF* in the solar atmosphere. Lines which may possibly be obscured or blended are marked "Absent?"

The percentage of unobscured lines identified is 82, much higher than would be expected by chance. The average Rowland intensity of the 28 lines identified is -1.0.

The strong atomic lines are so numerous in the region of the λ 4368.22 band that they completely obliterate the head and the more prominent features of the band structure. The region near the head of another strong band of *SiF* at λ 4400 is, however, unobscured, and here the correspondence in position and intensity between the solar and *SiF* lines is very marked.

Zirconium oxide.—Bands in the spectra of S-type stars have been identified by Merrill[1] with oxides of titanium and zirconium. Owing to the prominence of the *TiO* bands in the sun-spot spectrum, the *ZrO* bands might also be expected to be present. Search for the *ZrO* bands was made very difficult by the lack of wave-lengths and intensities for these bands measured on high-dispersion spectrograms. In order to procure material suited to this type of investigation, the strongest *ZrO* bands were photographed in the first order with the 75-foot spectrograph. The dispersion and resolution were sufficient to reveal the principal features of the bands, as well as the fine structure almost up to the band heads.

The *ZrO* bands apparently occur in two main groups: one in the blue-green spectral region and the other in the orange-red. All the bands observed are degraded to the red. The orange-red group includes the strongest bands, and since in the solar spectrum the number of atomic lines is much less in the orange-red than in the blue-green, it was decided to limit the investigation to the interval λ 5550 to λ 6500.

The wave-lengths and intensities of the band heads in the orange-red have been measured and are listed in Table III. The positions are referred to the iron-arc wave-lengths of St. John and Babcock,[2]

[1] *Publications of the Astronomical Society of the Pacific*, **35**, 217, 1923.

[2] *Mt. Wilson Contr.*, No. 202; *Astrophysical Journal*, **53**, 260, 1921.

TABLE II*a*

LINES OF INTENSITY 6, 7, 8, 9, 10 IN THE (0,0) λ 4368.22 BAND OF *SiF* COINCIDING WITH UNIDENTIFIED LINES IN THE *Revised Rowland Table*

Int. *SiF*	λ *SiF*	Int. Sun	λ Sun	*SiF*−Sun	Obscuring Line	Int.	El.*
8.....	4369.410	.....	Obscured?	.....	4369.408	1	*Fe*$^+$
6.....	4369.567	−1d?	4369.551	+0.016	.....	.....	.....
7.....	4369.650	.....	Obscured	.....	4369.716	0	*Ti*
7.....	4369.735	.....	Obscured	.....	4369.781	4	***Fe CH***
7.....	4369.822	−1	4369.868	− .046	.....	.....	.....
7.....	4369.909	−1	4369.868	+ .041	.....	.....	.....
7.....	4369.995	.....	Obscured	.....	4370.032	−1	*Ni*
7.....	4370.080	.....	Absent?	.....	.....	.....	.....
7.....	4370.175	0	4370.161	+ .014	.....	.....	.....
7.....	4370.264	0	4370.296	− .032	.....	.....	.....
7.....	4370.358	.....	Absent?	.....	.....	.....	.....
7.....	4370.454	−1	4370.426	+ .028	.....	.....	.....
7.....	4370.549	−2	4370.586	− .037	.....	.....	.....
6.....	4370.651	−1	4370.658	− .007	.....	.....	.....
6.....	4370.747	.....	Not observed	.....	.....	.....	.....
6.....	4370.851	−2d?	4370.859	− .008	.....	.....	.....
6.....	4372.384	0	4372.341	+ .043	.....	.....	.....
6.....	4372.505	−1	4372.499	+ .006	.....	.....	.....
6.....	4372.625	−3	4372.590	+ .035	.....	.....	.....
6.....	4372.743	.....	Obscured	.....	4372.742	0	*CH*
7.....	4372.861	0	4372.851	+ .010	.....	.....	.....
7.....	4372.989	.....	Obscured	.....	4372.993	0	*Fe*
7.....	4373.112	−2	4373.121	− .009	.....	.....	.....
7.....	4373.238	.....	Obscured	.....	4373.266	1	*V*−***Cr***
7.....	4373.365	−1N	4373.398	− .033	.....	.....	.....
7.....	4373.494	.....	Not observed	.....	.....	.....	.....
7.....	4373.626	.....	Obscured?	.....	4373.657	−3	*Cr*
7.....	4373.759	0	4373.792	− .033	.....	.....	.....
8.....	4373.892	0	4373.899	− .007	.....	.....	.....
8.....	4374.027	−2	4374.050	− .023	.....	.....	.....
8.....	4374.161	.....	Obscured	.....	4374.175	1	*Cr*
8.....	4374.285	.....	Absent?	.....	.....	.....	.....
8.....	4374.439	.....	Obscured	.....	4374.474	3	***Sc***$^+$*−Fe*
9.....	4374.581	−1N	4374.619	− .038	.....	.....	.....
10.....	4374.723	.....	Absent?	.....	4374.619	−1N	Unidentified
9.....	4374.869	.....	Obscured	.....	4374.825	0	***Ti***$^+$ *Rh*
9.....	4375.022	.....	Obscured	.....	4375.060	−2	*Nd*$^+$*−V*
9.....	4375.167	0Nd?	4375.202	− .035	.....	.....	.....
9.....	4375.317	.....	Obscured	.....	4375.337	0	*Cr*
9.....	4375.470	−1	4375.487	−0.017	.....	.....	.....

* Bold-face type indicates the dominating element of the blend.

TABLE II*a*—*Continued*

Int. *SiF*	λ *SiF*	Int. Sun	λ Sun	*SiF*−Sun	Obscuring Line	Int.	El.
9.....	4375.623	0	4375.578	+0.045	.....	.....	.....
9.....	4375.781	−3	4375.784	− .003	.....	.....	.....
9.....	4375.942	.....	Obscured	.....	4375.946	6	*Fe*
10.....	4376.103	.....	Obscured	.....	4376.946	6	*Fe*
8.....	4376.269	−1N	4376.225	+ .044	.....	.....	.....
10.....	4376.442	0N	4376.416	+ .026	.....	.....	.....
8.....	4376.609	0	4376.571	+ .038	.....	.....	.....
10.....	4376.781	.....	Obscured	.....	4376.784	1	*Fe*−*Cr*
8.....	4376.955	−1N	4376.957	− .002	.....	.....	.....
10.....	4377.138	−1N	4377.098	+0.040	.....	.....	.....

and band heads that are sharp and well defined should not be in error by more than 0.02 A. When the head is indistinct, however, the error may be 0.04 A or even higher. The exact position of a band head is hard to determine because the blackening on the photographic plate is all in one direction—to the red in the case of the

TABLE II*b*

SUMMARY OF DATA IN TABLE II*a*

Total number of *SiF* lines of intensity 6, 7, 8, 9, 10....	50
Blended or obscured by solar lines.....	16
Relatively unobscured.....	34
Identified.....	28
Apparently absent.....	2

ZrO bands. For this reason, the band heads will apparently be displaced to the violet side of their true positions. It is somewhat the same as if in measuring a line spectrum one always made settings on the violet side of a line instead of in the center. In order to correct for this "edge effect" an atomic line was selected which corresponded in blackness to that of some particular band head. The width of this line was determined in angstroms and the band head then shifted to the red by half this amount. No correction was made for some of the weaker bands. The first column in Table III gives the measured positions of the band edges; the second, the amount by which the wave-lengths in the first column should be increased to correct for the edge effect.

The intensities in the third column were obtained by comparing the blackness of the band heads with blackened strips of known in-

TABLE III

THE ORANGE-RED ZIRCONIUM OXIDE BAND HEADS

λ (Mt. W.)	Correction for Edge Effect	Intensity and Character	Probable Error	λ (Kiess) Bureau Standards	Mt. W.—Kiess
5551.61.....	+0.10	42 hs	II	5551.75	−0.04 A
5553.00.....	.07	60 hs	II	5553.17	.10
5579.99.....	.06	42 hnm	II	.....	.....
5581.41.....	.06	42 hnm	II	.....	.....
5609.93.....	.03	25 hnm	III	5610.18	.22
5628.84.....	.05	30 hvs	I	5629.02	.13
5629.36.....	.05	51 hvs	I	5629.58	.17
5633.69.....	.00	42 p	III	5633.98	.29
5717.97.....	.09	100 hvs	I	5718.21	.15
5723.70.....	.00	42 p	III	.....	.....
5747.96.....	.03	60 hs	III	5748.17	.18
5778.31.....	.03	20 hs	II	5778.57	.23
5835.00.....	.02	13 hnm	IV	.....	.....
5859.93.....	.03	42 hs	II	5860.14	.18
5908.40.....	.03	30 hs	II	5908.61	.18
6120.06.....	.02	13 hnm	II	.....	.....
6153.79.....	.02	13 hnm	II	6153.86	.05
6168.95.....	.02	10 hnm	IV	.....	.....
6170.21.....	.03	16	II	6170.49	.25
6203.98.....	.00	11 hnm	IV	6204.13	.15
6229.34.....	.11	110 hnm	I	6229.51	.06
6235.23.....	.00	25 p	III	.....	.....
6260.94.....	.07	55 h	III	6261.05	.04
6292.74.....	.07	50 h	III	6292.84	.03
6324.30.....	.03	30 hnm	II	6324.55	.22
6344.86.....	.09	120 hvs	I	6345.10	.15
6351.26.....	.00	30 p	III	.....	.....
6378.31.....	.05	80 h	III	6378.56	.20
6395.93.....	.00	13 hnm	IV	.....	.....
6412.25.....	.00	16 p	III	6412.39	.14
6473.42.....	.00	20 h?	I	.....	.....
6473.60.....	.06	100 hvs	I	6473.79	.13
6480.63.....	.00	20 p	III	6480.98	.35
6507.91.....	.04	50 h	III	6508.15	.20
6542.86.....	+0.00	25 hnm	IV	6542.90	−0.04

tensity-ratio on a photographic plate. More consistent intensities were probably obtained in this way than by simply making eye-estimates. The values were adjusted to give the weakest band an intensity of 10.

The symbols following the intensities indicate the character of the band head and the accuracy of the wave-length determinations.

h	= band head	I	= probable error of 0.00–0.01 A
hs	= head sharp	II	= probable error of 0.01–0.02
hvs	= head very sharp	III	= probable error of 0.02–0.03
hnm	= head not marked	IV	= probable error greater than 0.03
p	= part of band structure		

The last two columns compare the Mount Wilson wave-lengths with those of C. C. Kiess[1] measured at the Bureau of Standards in 1927. To photograph this region Kiess used an Anderson grating with 7500 lines per inch, which gave him a dispersion of 10.4 A per millimeter in the first order. A systematic difference Mount Wilson − Kiess = −0.16 A is indicated.

An examination of the spot spectrum at the positions of the stronger bands in Table III shows the λ 6474 band to be by far the most favorably placed for identification. Comparatively few atomic lines occur in this region, and the only molecular lines present arise from a weak *TiO* band at λ 6478. Besides occurring in an unobscured region, the λ 6474 band is the strongest in stars of class S. One might, therefore, expect it to be the most easily recognized *ZrO* band in the sun-spot spectrum.

In work of this sort a direct comparison of laboratory and stellar spectra, side by side, often gives as satisfactory an identification as does agreement in wave-length. The *ZrO* spectrograms were placed in a projecting machine and enlarged to the same scale as the Mount Wilson spot plates of this region. These plates were taken with the 75-foot spectrograph in the second order and have a dispersion of 1 A = 2.9 mm. When the two spectra were properly aligned, by matching iron-arc comparison lines with solar iron lines, the heads of the λ 6474 and λ 6481 bands coincided almost exactly with two faint lines in the spot spectrum.

The marking at λ 6474 differs considerably, however, from the appearance of this band head in the arc. In the spot, it consists of three faint lines, the one of shortest wave-length having the greatest intensity and the other two about equal intensity. In the arc, the

[1] *Bulletin of the Bureau of Standards*, **22**, 47, 1927.

head of the band is overexposed and resembles a broad black line shading off in intensity toward the red.

In order to make a closer comparison more spectrograms of the λ 6474 band were taken in the second order with the 75-foot spectrograph. The plates were slightly underexposed to prevent obliteration of lines very close to the head. These photographs clearly showed two lines near the head of the band. The wave-lengths of the three spot lines and of the band head and the two lines close to it are:

λ Arc	λ Spot	Arc−Spot	Remarks
6473.68.....	6473.70	−0.02 A	Head of 6474 band
6473.87.....	6473.83	+ .04	
6474.05.....	6473.97	+0.08	

When the faintness of these lines is considered, closer agreement in wave-length can hardly be expected. The agreement cannot be chance coincidence due to multiplicity of lines at the band head, for no other lines can be seen within about an angstrom of the head of the band on either the arc or the spot spectrograms.

There is evidence that the lines λ 6473.87 and λ 6474.05 are more persistent than the other lines in the band. When zirconium undergoes strong oxidation in the arc, the flame is a bright yellowish-white and the bands are very brilliant. When little or no oxidation is taking place, the flame is pale violet, fringed with green, and usually only the head of the band can be seen. Plates taken with sufficient exposure time, but when the flame was mostly violet in color, showed the lines λ 6473.87 and λ 6474.05 only slightly diminished in intensity. But other lines which appear much stronger than these two on plates taken when the arc was burning better were either absent or else very faint. Hence, if *ZrO* is scarce in sun-spots, the head of the band and the lines λ 6473.87 and λ 6474.05 should be the most conspicuous features of the λ 6474 band in the spot spectrum.

In addition to this triplet configuration at the head of the band, the strongest lines in the interval λ 6474 to λ 6508 were measured and searched for in the spot spectrum with the results given in Table IV. Owing to the weak and diffuse character of the spot lines, it was impossible to determine their wave-lengths very accurately.

For this reason they are given to only two decimal places. The arc intensities are on an arbitrary scale of 0–100. The wave-lengths and intensities of the obscuring lines are taken from the *Revised Rowland*

TABLE IV

LINES IN THE λ 6474 BAND OF *ZrO* IDENTIFIED WITH SOLAR LINES

Int. *ZrO*	λ *ZrO*	Int. Spot	λ Spot	*ZrO*−Spot	Obscuring Lines	Int. Spot	El.
100.....	6473.66	−2	6473.65	+0.01 A	.....	.....	.....
15.....	6480.64	−2	6480.62	+ .02	.....	.....	.....
20.....	6481.39	−3	6481.40	− .01	.....	.....	.....
15.....	6482.20	−3	6482.24	− .04	.....	.....	.....
15.....	6483.03	−2	6483.02*	+ .01	.....	.....	.....
15.....	6483.47	.....	Obscured	.....	6483.46	−3	Atm. wv.
15.....	6484.20	.....	Obscured	.....	6484.00†	1	.....
25.....	6484.39	−3	6484.42*	− .03	.....	.....	.....
15.....	6485.29	.....	Absent	.....	.....	.....	.....
15.....	6486.82	.....	Obscured	.....	6486.79	0	Atm. wv.
15.....	6487.86	−3	6487.84	+ .02	.....	.....	.....
20.....	6488.98	−2	6488.98	.00	.....	.....	.....
25.....	6490.63	−3	6490.56	+ .07	.....	.....	.....
15.....	6492.41	−3	6492.38	+ .03	.....	.....	.....
15.....	6493.80	.....	Obscured	.....	6493.80	12	*Ca*
15.....	6494.33	.....	Obscured	.....	6494.51	0	Atm. wv.
30.....	6495.36	−2	6495.32	+ .04	.....	.....	.....
30.....	6495.82	−2	6495.88	− .06	.....	.....	.....
25.....	6496.39	.....	Obscured	.....	6496.48	1	*Fe*
25.....	6496.94	.....	Obscured	.....	6496.92	6	*Ba*+
20.....	6498.52	−2	6498.44	+ .08	.....	.....	.....
25.....	6499.11	.....	Obscured	.....	6499.66	8	*Ca*
20.....	6499.78	.....	Obscured	.....	6499.66	8	*Ca*
20.....	6500.37	−2	6500.34	+ .03	.....	.....	.....
25.....	6505.46	−3	6505.53	− .07	.....	.....	.....
15.....	6506.92	.....	Absent	.....	.....	.....	.....
20.....	6507.58	−3	6507.61*	−0.03	.....	.....	.....
50.....	6507.91	.....	Obscured?	.....	6508.16	2	Atm. *Ti*

* It is believed that lines in the *Revised Rowland Table* at these wave-lengths have been incorrectly attributed to water vapor. They are intensified in the spot spectrum but not in the disk spectrum on a plate taken soon after sunrise.

† An atomic line appears in the spot spectrum at this wave-length. It is not given in the *Revised Rowland Table.*

Table. The spot wave-lengths have been diminished by 0.04 A to make the sum of the differences in the fifth column equal to zero.

Only two of the weakest band lines in Table IV are apparently absent from the spot, and, on the whole, the evidence for the presence of *ZrO* in sun-spots as presented in this table is rather favorable.

The final conclusion drawn from the investigation of the three lines at the band head and from the strongest lines in the band is that very weak lines of the λ 6474 band of *ZrO* are present in the sun-spot spectrum.

Aluminum oxide.—F. E. Baxandall[1] has identified several bands in the spectrum of Mira Ceti with *AlO*.[2] The blue-green bands (0,0) λ 4842, (1,0) λ 4648, and (0,1) λ 5079, all degraded to the red, are the strongest in the *AlO* spectrum, having relative intensities about 8, 7, and 5, respectively. The λ 4842 and λ 4648 bands are those most favorably placed for identification. *TiO* bands at λ 4629, λ 4669, and λ 4848, for which A. Fowler[3] gives intensities of 4, 4, and 3, respectively, on a scale of 10, are probably too weak to obscure the *AlO* bands very seriously. The identification of lines in the λ 4842 band was not attempted beyond the head of the strong (1,0) λ 4954 band of *TiO*. The λ 5079 band, besides being the weakest of the three, is intermingled with the Swan bands at λ 5096 and λ 5129, which makes its identification exceptionally difficult.

The wave-lengths of the *AlO* lines were obtained from spectrograms taken by R. T. Birge[4] in the second order of the concave grating of the University of Wisconsin, with a dispersion of 1.35 A per millimeter. For distinct lines they are considered accurate within about 0.005 A. The solar wave-lengths were for the most part taken from the list of unpublished measurements referred to in section 2, but a few are taken from the *Revised Rowland Table*. It was decided to consider lines coincident when they did not differ in wave-length by more than 0.035 A. Unfortunately, arc intensities of the *AlO* lines were not available; but since the lines must be very weak in the spot spectrum, only spot lines of intensity 0 or less were considered for identification.

The data are too extensive to publish in detail, as was done for the *SiF* and *ZrO* bands, but the final results of the comparison are

[1] *Monthly Notices of the Royal Astronomical Society*, **88**, 679, 1928.

[2] Joy (*Mt. Wilson Contr.*, No. 311; *Astrophysical Journal*, **63**, 281, 1926) records two unidentified bands in the spectrum of Mira Ceti at approximately λ 4243 and λ 4331, which Baxandall does not mention in his paper. These are probably the (8,4) λ 4243 and (4,1) λ 4330 bands of *AlO*.

[3] *Proceedings of the Royal Society of London*, A, **79**, 509, 1907.

[4] *Physical Review*, **29**, 59, 1927.

given in Table V. In view of the faintness of these bands in the spot spectrum compared with those of *CaH*, *MgH*, C_2, and *TiO*, the percentage of lines satisfying the requirement specified in section 4 is quite satisfactory. Since *AlO* bands are already known to exist in Mira Ceti, and since none of the 141 lines of Table V had previously been identified, the observational evidence for the presence of *AlO* in sun-spots is very good. Fifty-two per cent, or 150, of the unobscured lines could not be identified. Only the strongest of the *AlO* lines, however, can be expected to show in a spectrum having such large irregularities in background intensity as that of sun-spots. These

TABLE V

SUMMARY OF RESULTS FOR THE *AlO* BANDS

	λ 4842	λ 4648	λ 5079	Totals and Means
Total number of lines	192	122	60	374
Obscured	44	30	9	83
Unobscured	148	92	51	291
Identified	71	48	22	141
Percentage identified	48	52	43	48
Chance coincidences	30	10	5	45
Percentage chance coincidences	20	11	10	14
Average difference in wave-length	0.016 A	0.016	0.015	0.016 A

missing lines are probably either too faint to be seen at all or else are obliterated by extensively winged lines.

The number of chance coincidences in Table V has been computed from the formula of Russell and Bowen,[1]

$$C = N[1 - \exp.\ (-2x/I)]\ , \qquad (1)$$

where C is the average number of chance coincidences, N the number of lines in the region under investigation that are available for identification, x the largest difference permissible (0.035 A in this case), and I the average interval between adjacent lines of *AlO*. An actual count shows that for the λ 4842 band, in the interval λ 4842 to λ 4954, $N = 335$, $I = 112/148 = 0.76$. In computing C for the λ 4648 and λ 5079 bands, it was assumed that the number of lines available for identification per angstrom is the same for these bands as for the λ 4842 band.

[1] *Mt. Wilson Contr.*, No. 375; *Astrophysical Journal*, **69**, 196, 1929.

Aluminum hydride.—If *AlO* is present in the solar atmosphere, it is quite likely that *AlH* is there also.[1] The spectra of sun-spots and the reversing layer were searched for the *AlH* bands with heads at λ 4546, λ 4671, and λ 4752. It was necessary to exclude the λ 4241 band from investigation, since spot wave-lengths were not available for this region. Bengtsson and Rydberg[2] have published the intensities and the wave-numbers to one decimal for the lines in these bands. Until more accurate measurements are published, the identification of solar lines with those of *AlH* is rather uncertain. For this reason the present investigation can be regarded only as preliminary. On the basis of the present material, however, there is good evidence for the presence of *AlH* bands in the spectra of sun-spots and the reversing layer. The percentage of lines identified is much higher than

TABLE VI

SUMMARY OF RESULTS FOR THE *AlH* BANDS

Intensities of *AlH* lines	0, 0.5	1, 1.5	2, 2.5	3, 3.5, 4
No. lines unobscured	8	20	42	21
Identified	4	12	29	15
Percentage identified	50	60	69	71
Mean Rowland intensity	−2.8	−1.9	−1.5	−1.5

the percentage of chance coincidences, and the laboratory and solar intensities are in close agreement. Only a brief summary of the results can be given here (Table VI).

Secondary spectrum of hydrogen.—The earliest attempt to detect the presence of the secondary spectrum of hydrogen in the sun-spot spectrum was made by T. R. Merton[3] in 1922. He found comparatively few coincidences between spot and laboratory lines, and, since there was no correspondence in intensity, concluded that molecular hydrogen did not exist in sun-spots. Seven years later G. Piccardi[4] reported the discovery of the secondary spectrum of hydrogen in the sun-spot spectrum. His laboratory data were obtained from the extensive list of H_2 lines measured by Gale, Monk,

[1] The search for the *AlH* bands was undertaken at the suggestion of Dr. H. N. Russell, who predicted from theory their presence in the solar spectrum.

[2] *Zeitschrift für Physik*, **59**, 540, 1929.

[3] *Philosophical Transactions of the Royal Society of London*, A, **222**, 369, 1922.

[4] *Società Astronomica Italiana, Memorie*, **4**, 379, 1929.

and Lee,[1] and the spot wave-lengths were read from the Mount Wilson map of the spot spectrum to the nearest 0.01 A. Using only lines of intensity 7, 8, 9, and 10 on a scale of 10, Piccardi was able to identify about 75 per cent of the H_2 lines in the spectrum from λ 4486 to λ 6428. His maximum difference in wave-length is 0.03 A.

The writer has compared his wave-length and intensity data from λ 4500 to λ 5500 with those of Gale, Monk, and Lee for this region. The results are given in Table VII.

The percentage of H_2 lines identified having intensities 7, 8, and 9 is about the same as the percentage of chance coincidences. But the very high percentage of identifications for lines of intensity 10 indicates that some of these may be real. If so, only the very strongest

TABLE VII

SUMMARY OF RESULTS FOR THE SECONDARY SPECTRUM OF HYDROGEN

Intensities of H_2 lines	7	8	9	10
No. lines unobscured	8	16	11	14
Identified	3	7	4	12
Percentage identified	38	44	36	86
Mean Rowland intensity	−1.1	−1.8	−1.6	−2.0

lines of molecular hydrogen that appear in laboratory sources are of sufficient intensity to be observed in the sun-spot spectrum. The writer believes, however, that on the basis of the evidence in Table VII alone it would be imprudent to make a positive statement that molecular hydrogen is present in sun-spots.

Yttrium.—C. D. Shane[2] has tentatively attributed certain bands in the spectrum of Mira Ceti to some compound of yttrium, probably the oxide. No certain indications of flutings were found at the positions of the stronger yttrium bands given by Meggers and Wheeler.[3]

Scandium oxide.—Baxandall[4] has identified several bands in the spectrum of Mira Ceti with *ScO*. Meggers and Wheeler[3] have measured the positions of the *ScO* band heads and estimated their inten-

[1] *Astrophysical Journal*, **67**, 89, 1928.

[2] *Lick Observatory Bulletins*, **10**, 131, 1918–1923.

[3] *Bureau of Standards Journal of Research*, **6**, 239, 1931.

[4] *Publications of the Astronomical Society of the Pacific*, **41**, 168, 1929.

sities. No band heads could be found in the spot spectrum at the wave-lengths of the strongest *ScO* flutings. Since the region of several of these flutings was unobscured, the band should easily have been seen if there. Hence, *ScO* seems definitely to be absent from the sun-spot spectrum.

Evidence for the existence of MgH and TiO in the reversing layer.—It is interesting to ascertain whether the bands of *MgH* and *TiO* appear faintly in the spectrum of the reversing layer as well as in that of the sun-spot. Fowler,[1] Evershed,[2] and Hale and Adams,[3] all concluded that *MgH* is probably present in the reversing layer, but apparently found no evidence for the presence of *TiO*. The spot

TABLE VIII

RESULTS OF SEARCH FOR *MgH* AND *TiO* IN THE REVERSING LAYER

	TiO		*MgH*
	λ 4954	λ 5167	λ 5211
Total number lines identified	131	127	105
Observed in disk	23	24	60
Found in *Revised Rowland Table*	22	17	50
Percentage found in disk	17	16	52
Percentage chance coincidences	18	17	18

lines attributed to *MgH* and *TiO* in this investigation have been examined to see if they could be found in the disk. The results are given in Table VIII. *TiO* lines blended with *MgH* were excluded and also *MgH* lines blended with those of C_2. *MgH* seems definitely to be present in the reversing layer, in agreement with the results of earlier investigators. The percentage of *TiO* lines found in the disk is much less than that of *MgH* and agreements in position are probably all chance coincidences. *TiO* is no doubt confined entirely to the sun-spot.

Presence of both TiO and C_2 bands in the sun-spot spectrum.—No star has yet been discovered in which the *TiO* bands and the Swan bands of carbon, C_2, are both present—the two appear to be mutual-

[1] *Philosophical Transactions of the Royal Society of London*, A, **209**, 47, 1909.

[2] *Memoirs of the Kodaikanal Observatory*, **1**, 1, 1909.

[3] *Mt. Wilson Contr.*, No. 40; *Astrophysical Journal*, **30**, 86, 1909.

ly exclusive. The Swan bands first appear in the spectra of Ro stars[1] and attain a maximum intensity at R5, followed by a minimum at No with a subsequent rise. Bands of *TiO* are first observed at about class K5 and increase steadily in intensity to M8. The stellar sequence seems to branch at about class K3. The majority of stars are of types K and M, but a few go through some evolutionary process as R and N stars.

In view of these facts pertaining to late-type stars, it is worth while calling attention to the circumstance that both the bands of *TiO* and C_2 appear in the sun-spot spectrum (stellar class about Ko) in about equal intensity. This is the only known exception to the statement that bands of carbon and *TiO* are never observed together in the same star. It must be remembered, however, that these bands might be found in the same star if stellar spectra could be photographed with dispersion as high as that used in photographing the solar spectrum.

6. THEORY FOR DETERMINING TEMPERATURES FROM THE DISTRIBUTION OF LINE INTENSITIES IN BAND SPECTRA

On the basis of the quantum theory the intensities of the absorption lines of any particular band arising from the vibrational transition $v''-v'$ will be proportional to the product of the number of molecules in the lower energy state and the probability of a transition from this state to a higher energy level. Thus for thermal equilibrium the relative intensities, I, of the lines in an absorption band can be determined by an equation of the form[2]

$$I = i \exp.\ (-E''/kT)\ , \qquad (2)$$

where $i = a\bar{p}$. The other symbols in equation (2) have the following significance: E'' is the rotational energy in the lower molecular state and equals $h^2J''^2/8\pi^2I''$, where $J''h/2\pi$ is the resultant angular momentum of the molecule in the lower energy state, and I'' is the moment of inertia of the molecule in the lower energy state. This is not the entire expression for the rotational energy of the molecule, but it is sufficiently accurate for the purpose of this investigation. $\bar{p}$ is the

[1] *Lick Observatory Bulletins*, **13**, 123, 1928.

[2] *Physical Review*, **25**, 1, 1925.

TABLE IX

BANDS DEFINITELY OR TENTATIVELY IDENTIFIED IN CELESTIAL SPECTRA

NAME OF BAND	CARRIER	SHADED	PROMINENT BAND HEADS (λ I.A.)	OCCURRENCE		LITERATURE REFERENCES	NOTES
				Stars and Comets	Sun		
Aluminum hydride	*AlH*	R	4241 4546 4752	.....	Disk? Spot?	1, 2	
Calcium hydride: B	*CaH*	V	6382.04	.....	Spot	3, 4	
B	.....	.....	6389.12				
A	.....	.....	6902.62				
C+H (Fraunhofer's G Band)	*CH*	V	4314.10	Types F5, G, K, M, N, R Comets	Disk Spot	5, 6, 7, 8	(1)
Raffety bands	*CH?*	V	4042.4 4052.1 4014.3	Comets		9, 10, 11	(1)
	CN?		4074.2		Disk		
Magnesium hydride	*MgH*	V	5621.35 5210.95 4844.74	*o* Ceti ?	Spot	12, 13, 14, 15	
Nitrogen hydride *β* (ammonia)	*NH*	R	3360.04	.....	Disk Spot	16	
Water vapor	*OH*	R	3063.71	.....	Disk Spot	17	
Silicon hydride	*SiH*	R	4143.89	.....	Spot	18, 19	(2)
Aluminum oxide	*AlO*	R	4647.96 4842.26 5079.34	*o* Ceti	Spot	20, 21, 22	(3)
Boron monoxide (*α*-group)	$B^{11}O$	R	4363.4 4612.7 5040.1	.....	Spot	23, 24	(4)
Comet tail (third negative carbon group)	CO^{+}	R	4018.68 4272.59 4567.27	Comets	.....	11, 25, 26, 27, 28, 29	(5)
Scandium oxide	*ScO*	R	4857.78 5847.73 6036.17	*o* Ceti	.....	30, 31, 32	
Titanium oxide	*TiO*	R	4954.55 5166.86 7054.39	Types M, S	Spot	33, 34, 35, 36, 37	

TABLE IX—*Continued*

Name of Band	Carrier	Shaded	Prominent Band Heads (λ I.A.)	Occurrence		Literature References	Notes
				Stars	Sun		
Yttrium	*YO* ?	R	4818.22 5972.10 6132.13	*o* Ceti ?	.....	31, 38, 39	
Zirconium oxide	*ZrO* ?	R	6229.45 6344.95 6473.66	Type S	Spot	40, 41, 42, 43	
Swan	C_2	V	4737.08 5165.22 5635.53	Types R, N Comets	Disk Spot	11, 29, 44, 45, 46, 47, 48, 49, 50	(6)
Violet cyanogen	*CN*	V	3590.33 3883.40 4215.96	Types G, K, M, N, R Comets	Disk Spot	11, 29, 46, 49, 51, 52	(7)
Second negative group of nitrogen	N_2^+	V	3914.34 4278.11 4709.33	Comets	.....	11, 25, 26, 27, 29, 53	(8)
Hydrogen: α (Fulcher) α (Fulcher) β	H_2	R	6018.29 6327.07 5760.37	.....	Spot?	54, 55, 56, 57	
Silicon fluoride α system	$Si^{28}F$?	R	4368.22 4400.50 4430.23	.....	Disk Spot	58	

NOTES TO TABLE IX

1. The Raffety bands and the λ 4314 hydrocarbon band are observed in emission principally in the nucleus and head of comets.
2. The reproduction of the *SiH* band spectrum published by C. V. Jackson (Ref. 18, p. 392) shows no marked heads.
3. Emission bands of *AlO* were observed in the spectrum of *o* Ceti at the exceptionally low maximum of February, 1924. Absorption bands of *AlO* probably exist in the normal spectrum.
4. These wave-lengths are given by R. S. Mulliken (Ref. 23). Spectrograms of boric acid taken by King show no distinct band heads at these positions.
5. The spectrum of the comet tail shows principally the bands of CO^+ in emission.
6. In comets, emission bands of the Swan spectrum occur chiefly in the nucleus and coma; in some cases they probably exist in the tail as well.
7. Emission bands of *CN* are observed in the spectra of the nucleus and coma of comets; their existence in the tail has not been proved.
8. Emission bands of N_2^+ are observed in the spectra of comets' tails.

LITERATURE REFERENCES FOR TABLE IX

1. Eriksson and Hulthén, "Über die Bandenspektra von Aluminium," *Zeitschrift für Physik*, **34**, 775, 1925.
2. Bengtsson and Rydberg, "Die Bandenspektra von Aluminiumhydrid," *ibid.*, **59**, 540, 1929.
3. Olmsted, "Sun-Spot Bands Which Appear in the Spectrum of a Calcium Arc Burning in the Presence of Hydrogen," *Mt. Wilson Contr.*, No. 21; *Astrophysical Journal*, **27**, 66, 1908.
4. Hulthén, "On the Band Spectrum of Calcium Hydride," *Physical Review*, **29**, 97, 1927.
5. Eder and Valenta, "Spectralanalyse der Leuchtgasflamme," *Beiträge zur Photochemie und Spectralanalyse*, p. 345, 1904.
6. Newall, Baxandall, and Butler, "On the Identity of Fraunhofer's Group G in the Solar Spectrum with the Hydrocarbon Band 4314," *Monthly Notices of the Royal Astronomical Society*, **76**, 640, 1916.
7. Heurlinger and Hulthén, "Über die Struktur des Bandenspektrums brennender Kohlenwasserstoffe," *Zeitschrift für Wissenschaftliche Photographie*, **18**, 241, 1919.
8. Kratzer, "Die Terme der $(C+H)$-Banden," *Zeitschrift für Physik*, **23**, 298, 1924.
9. Raffety, "On Some Investigations of the Spectra of Carbon and Hydrocarbon," *Philosophical Magazine*, **32**, 546, 1916.
10. Baldet, "Comparaison de plusieurs radiations émises par le noyau des comètes, et d'origine encore inconnue, avec le spectre du bec Mecker," *Comptes rendus*, **177**, 1205, 1923.
11. Bobrovnikoff, "The Raffety Bands in Comets," *Astrophysical Journal*, **73**, 61, 1931.
12. Fowler, "The Spectrum of Magnesium Hydride," *Philosophical Transactions of the Royal Society of London*, A, **209**, 447, 1909.
13. Watson and Rudnick, "The Magnesium Hydride Band Spectrum," *Astrophysical Journal*, **63**, 20, 1926.
14. Joy, "A Spectrographic Study of Mira Ceti," *Mt. Wilson Contr.*, No. 311; *Astrophysical Journal*, **63**, 281, 1926.
15. Watson and Rudnick, "Rotational Terms in the *MgH* Bands," *Physical Review*, **29**, 413, 1927.
16. Fowler and Gregory, "The Ultra-Violet Band of Ammonia, and Its Occurrence in the Solar Spectrum," *Philosophical Transactions of the Royal Society of London*, A, **218**, 351, 1919.
17. Fowler, "The Presence in the Solar Spectrum of the Water-Vapour Band λ 3064," *Proceedings of the Royal Society of London*, A, **94**, 472, 1918.
18. Jackson, "The Spectrum of Silicon Hydride," *ibid.*, **126**, 373, 1930.
19. Pearse, *unpublished*.
20. Hasselberg, "Zur Spectroscopie der Verbindungen. Spektrum der Thonerde," *Svenska Vetenskapsakademien Handlingar*, N.S., **24**, No. 15, 1891.
21. Pomeroy, "The Quantum Analysis of the Band Spectrum of Aluminum Oxide (λ 5200–λ 4650)," *Physical Review*, **29**, 59, 1927.
22. Baxandall, "On the Presence of Radiation Bands of Aluminium Oxide in the Spectrum of Mira Ceti at the Maximum of 1924, February," *Monthly Notices of the Royal Astronomical Society*, **88**, 679, 1928.
23. Mulliken, "The Isotope Effect in Band Spectra, II: The Spectrum of Boron Monoxide," *Physical Review*, **25**, 259, 1925.
24. Nicholson and Perrakis, "Evidence of Boron in the Sun," *Mt. Wilson Contr.*, No. 370; *Astrophysical Journal*, **68**, 327, 1928.

25. Fowler, "Terrestrial Reproduction of the Spectra of the Tails of Recent Comets," *Monthly Notices of the Royal Astronomical Society*, **70**, 176, 1909.
26. Fowler, "Investigations Relating to the Spectra of Comets," *ibid.*, p. 484, 1910.
27. Pluvinel and Baldet, "Spectrum of Comet Morehouse (1908*c*)," *Astrophysical Journal*, **34**, 89, 1911.
28. Merton, "On Spectra Associated with Carbon," *Proceedings of the Royal Society of London*, A, **103**, 383, 1923.
29. Baldet, "Recherches sur la Constitution des Comètes et sur les Spectres du Carbone," *Annales de l'Observatoire d'Astronomie physique de Paris (Mendon)*, **7**, 1926.
30. Fowler, "The Spectrum of Scandium, and Its Relation to Solar Spectra," *Philosophical Transactions of the Royal Society of London*, A, **209**, 47, 1909.
31. Meggers and Wheeler, "The Band Spectra of Scandium-, Yttrium-, and Lanthanum Monoxides," *Bureau of Standards, Journal of Research*, **6**, 239, 1931.
32. Baxandall, "Absorption Bands of Scandium Oxide in the Spectrum of *o* Ceti," *Publications of the Astronomical Society of the Pacific*, **41**, 168, 1929.
33. Fowler, "The Spectra of Antarian Stars in Relation to the Fluted Spectrum of Titanium," *Proceedings of the Royal Society of London*, **73**, 219, 1904.
34. Fowler, "The Fluted Spectrum of Titanium Oxide," *ibid.*, A, **79**, 509, 1907.
35. Hale and Adams, "Second Paper on the Cause of the Characteristic Phenomena of Sun-Spot Spectra," *Mt. Wilson Contr.*, No. 15; *Astrophysical Journal*, **25**, 75, 1907.
36. Christy, "Quantum Analysis of the Blue-Green Bands of Titanium Oxide," *Physical Review*, **33**, 701, 1929.
37. Christy, "New Band System of Titanium Oxide," *Astrophysical Journal*, **70**, 1, 1929.
38. Kayser, *Handbuch der Spectroscopie*, **6**, 823, 1921.
39. Shane, "The Spectrum of *o* Ceti in 1919," *Lick Observatory Bulletins*, **10**, 131, 1922.
40. Merrill, "Stellar Spectra of Class S: I. General Description," *Mt. Wilson Contr.*, No. 252; *Astrophysical Journal*, **56**, 457, 1922.
41. Merrill, "Zirconium Bands in Stellar Spectra of Class S," *Publications of the Astronomical Society of the Pacific*, **35**, 217, 1923.
42. King, "The Relative Temperatures Required To Produce the Oxide Bands of Zirconium and Titanium," *ibid.*, **36**, 140, 1924.
43. Kiess, "Wave-Length Measurements in the Arc and Spark Spectra of Zirconium," *Bulletin of the Bureau of Standards*, **22**, 47, 1927.
44. Hale, "On the Presence of Carbon in the Chromosphere," *Astrophysical Journal*, **6**, 412, 1897.
45. Hale, Ellerman, and Parkhurst, "The Spectra of Stars of Secchi's Fourth Type," *Yerkes Publications*, **2**, 253, 1903.
46. Shane, "The Spectra of Certain Class N Stars," *Lick Observatory Bulletins*, **10**, 79, 1920.
47. Johnson, "The Structure and Origin of the Swan Band Spectrum of Carbon," *Philosophical Transactions of the Royal Society of London*, A, **226**, 157, 1926.
48. Shea, "The Structure of the Swan Bands," *Physical Review*, **30**, 825, 1927.
49. Shane, "The Spectra of the Carbon Stars," *Lick Observatory Bulletins*, **13**, 123, 1928.
50. King and Birge, "Evidence from Band Spectra of the Existence of a Carbon Isotope of Mass 13," *Mt. Wilson Contr.*, No. 402; *Astrophysical Journal*, **72**, 19, 1930.
51. Kayser, *Handbuch der Spectroscopie*, **5**, 190, 1910.

52. Uhler and Patterson, "The Structure of the Third Cyanogen Band and the Associated Tails," *Astrophysical Journal*, **42**, 434, 1915.
53. Fassbender, "Untersuchungen über das negative Stickstoffbandenspektrum," *Zeitschrift für Physik*, **30**, 73, 1924.
54. Merton and Barratt, "On the Spectrum of Hydrogen," *Philosophical Transactions of the Royal Society of London*, A, **222**, 369, 1922.
55. McLennan, Grayson-Smith, and Collins, "Intensities in the Secondary Spectrum of Hydrogen at Various Temperatures," *Proceedings of the Royal Society of London*, A, **116**, 277, 1927.
56. Gale, Monk, and Lee, "Wave-Lengths in the Secondary Spectrum of Hydrogen," *Astrophysical Journal*, **67**, 89, 1928.
57. Piccardi, "Sulla presenza dell'idrogeno molecolare nelle macchie del sole," *Memorie della Società Astronomica Italiana*, **4**, 379, 1929.
58. Johnson and Jenkins, "The Band Spectrum of Silicon Fluoride," *Proceedings of the Royal Society of London*, A, **116**, 327, 1927.

mean of the a priori probabilities, p, of the upper and lower rotational energy states. a is the factor expressing the probability that any molecule in the lower energy state will make a transition to the higher energy state in a unit time. It is considered a constant in this paper, but it probably changes slowly with increasing values of J. k is Boltzmann's constant. T is the absolute temperature.

Hönl and London,[1] by an application of the summation rule for band spectra, have derived expressions[2] for i as a function of J and a quantity Ω. The latter is a quantum number correlated with the resultant electronic orbital angular momentum of the molecule parallel to the line joining the two nuclei. This application of the summation rule depends upon the existence of a close analogy between the expressions for the relative amplitudes of the frequency components corresponding to $\Delta J = 0$ (Q branch) and $\Delta J = \pm 1$ (R and P branches) in band spectra, and those for $\Delta j = 0, \pm 1$ in a line spectrum multiplet, where j is the inner quantum number; and for $\Delta\Omega = 0, \pm 1$ in band spectra, and $\Delta k = 0, \pm 1$ in atomic spectra, k being the azimuthal quantum number. These equations make it possible to calculate the relative intensities of band lines when the temperature and the values of Ω' and Ω'' for the upper and lower energy levels are known. Hönl and London's expressions for the P,

[1] *Zeitschrift für Physik*, **33**, 803, 1925.

[2] The notation of this paper is that given by Mulliken in his "Report on Notation for Spectra of Diatomic Molecules," *Physical Review*, **36**, 611, 1930.

Q, and R branches of a band are given below.[1] $\bar{J}$ is the mean of J' and J''. For the case $\Omega'=\Omega''=\Omega$:

$$\left.\begin{aligned} i_P &= i_R = 2a(\bar{J}^2-\Omega^2)/\bar{J}\ , \\ i_Q &= 4aJ\Omega^2/(J^2-\tfrac{1}{4})\ . \end{aligned}\right\} \tag{3}$$

For the case $\Omega'-\Omega''=+1$:

$$\left.\begin{aligned} i_P &= a(\bar{J}-\Omega')(\bar{J}-\Omega'+1)/\bar{J}\ , \\ i_Q &= 2aJ(J-\tfrac{1}{2}+\Omega')(J+\tfrac{1}{2}-\Omega')/(J^2-\tfrac{1}{4})\ , \\ i_R &= a(\bar{J}+\Omega')(\bar{J}+\Omega'-1)/\bar{J}\ . \end{aligned}\right\} \tag{4}$$

For the case $\Omega'-\Omega''=-1$:

$$\left.\begin{aligned} i_P &= a(\bar{J}+\Omega'')(\bar{J}+\Omega''-1)/\bar{J}\ , \\ i_Q &= 2aJ(J-\tfrac{1}{2}+\Omega'')(J+\tfrac{1}{2}-\Omega'')/(J^2-\tfrac{1}{4})\ , \\ i_R &= a(\bar{J}-\Omega'')(\bar{J}-\Omega''+1)/\bar{J}\ . \end{aligned}\right\} \tag{5}$$

Mulliken[1] has tested the intensity factors of Hönl and London for bands involving singlet electronic states. It was necessary for him to assume the temperature, and his observed intensities were only approximate; but, in general, the theoretical and experimental intensity-curves were in essential agreement. He concluded that equations (3), (4), and (5) should hold for doublet and triplet as well as singlet electron levels, and in a later paper[2] has stated that, in particular, they should be at least qualitatively applicable to the Swan bands, especially for low values of $\bar{J}$. Since, owing to the roughness of the observational material, a rigorous test of these equations is not possible here, the theoretical intensities calculated for the Swan bands have been used directly and without correction of any sort.

In addition to this work of Mulliken's, several other investigators[3] have determined effective temperatures of laboratory sources from the distribution of line intensities in band spectra. The only application of the quantum theory of band spectra to the determination of

[1] *Physical Review*, **29**, 391, 1927.

[2] *Ibid.*, p. 637, 1927.

[3] *Zeitschrift für Physik*, **49**, 315, 1928; **54**, 246, 1929; **64**, 443, 1930.

stellar temperatures, however, was that made in 1922 by Birge,[1] who obtained a temperature of $4000° \pm 500°$ C for the reversing layer from the position of maximum intensity in the λ 3883 *CN* band.

7. APPLICATION OF THE THEORY TO SUN-SPOTS AND THE REVERSING LAYER

The wave-length and intensity data referred to in section 2 are applicable to the problem of determining temperatures from band-line intensities, and have been used to find the temperatures of sun-spots and the reversing layer. The observational material was treated in the following manner:

The λ 5211 and λ 5167 bands of *MgH* and *TiO* proved to be so badly blended in sun-spots that it was impossible to ascertain the true distribution of intensity in them. The Swan band at λ 5165 is, however, relatively free from blends with other bands and enough lines could be found to show the true intensity distribution in the R branch, both in sun-spots and in the reversing layer. The P branch could not be used, since the lines are all near the band head and most of them blended with each other or with lines of *MgH*. No Q branch has been observed in the Swan bands.

Table X gives the lines that were selected. The solar wave-lengths have been adjusted to make the sum of the residuals zero. These lines satisfy the following requirements:

a) The laboratory and spot wave-lengths were found to differ systematically in the sense: Spot − Arc = +0.022 A. All the lines in Table X show a difference of about this amount.

b) No difference in the table is greater than 0.025 A; the average is 0.010 A.

c) None of the lines shows the Zeeman effect.

d) All the lines are intensified in passing from the disk to the spot.

e) As far as could be ascertained, the lines are unblended. The table itself shows how the data were combined to form the graphs in Figure 1.

From their close analogy with the second positive nitrogen bands, the Swan bands have been classified as $^3\Pi \leftarrow {}^3\Pi$. For a $^3\Pi$ state the Ω in (3), (4), and (5) is equal to $\Omega' = \Omega'' = \Omega = 0, 1, 2$. (In practice,

[1] *Astrophysical Journal*, **55**, 273, 1922.

TABLE X

LINES USED TO DETERMINE DISTRIBUTION IN THE λ 5165 SWAN BAND

λ Spot	λ C_2	Spot − C_2	J	Intensities	
				Spot	Disk
5153.066.....	5153.049	+0.017	5	−1.8	−3.0
5151.134.....	5151.143	− .009	7	−1.4	−3.0
5143.304.....	5143.307	− .003	12	−2.2	−2.8
Means...	.....	.....	8.0	−1.80	−2.92
5140.359.....	5140.363	− .004	14	−1.7	−2.9
5138.484.....	5138.493	− .009	15	−2.1	−2.9
5136.452.....	5136.448	+ .004	16	−1.9	−2.5
Means...	.....	.....	15.0	−1.90	−2.77
5134.292.....	5134.300	− .008	17	−2.1	−2.8
5125.957.....	5125.961	− .004	21	−2.0	−2.8
5123.874.....	5123.865	+ .009	22	−1.8	−3.0
Means...	.....	.....	20.0	−1.97	−2.87
5121.442.....	5121.424	+ .018	23	−1.4	−2.8
5116.883.....	5116.876	+ .007	25	−2.1	−2.8
5111.727.....	5111.703	+ .024	27	−1.2	−2.8
Means...	.....	.....	25.0	−1.57	−2.80
5109.187.....	5109.192	− .005	28	−1.4	−2.4
5106.406.....	5106.391	+ .015	29	−1.6	−2.6
5103.824.....	5103.831	− .007	30	−1.4	−2.2
Means...	.....	.....	29.0	−1.47	−2.40
5100.835.....	5100.857	− .022	31	−2.0	−2.1
5089.359.....	5089.350	+ .009	35	−0.9	−2.7
5083.036.....	5083.051	− .015	37	−1.5	−2.5
Means...	.....	.....	34.3	−1.47	−2.43
5080.089.....	5080.085	+ .004	38	−2.1	−2.8
5073.568.....	5073.578	− .010	40	−1.2	−2.2
5066.824.....	5066.845	− .021	42	−1.5	−2.1
Means...	.....	.....	40.0	−1.60	−2.37
5033.788.....	5033.774	+ .014	51	−1.8	−2.5
5005.494.....	5005.486	+ .008	58	−1.6	−3.0
4996.948.....	4996.925	+ .023	60	−2.2	−3.0
Means...	.....	.....	56.3	−1.87	−2.83
4983.513.....	4983.517	−0.004	63	−2.3	−3.0

only $\Omega=0$ was used, since the three values give almost identical values of I.) The constant factor $2a$ was taken as unity since relative intensities only were required. The intensity factor for i_R then

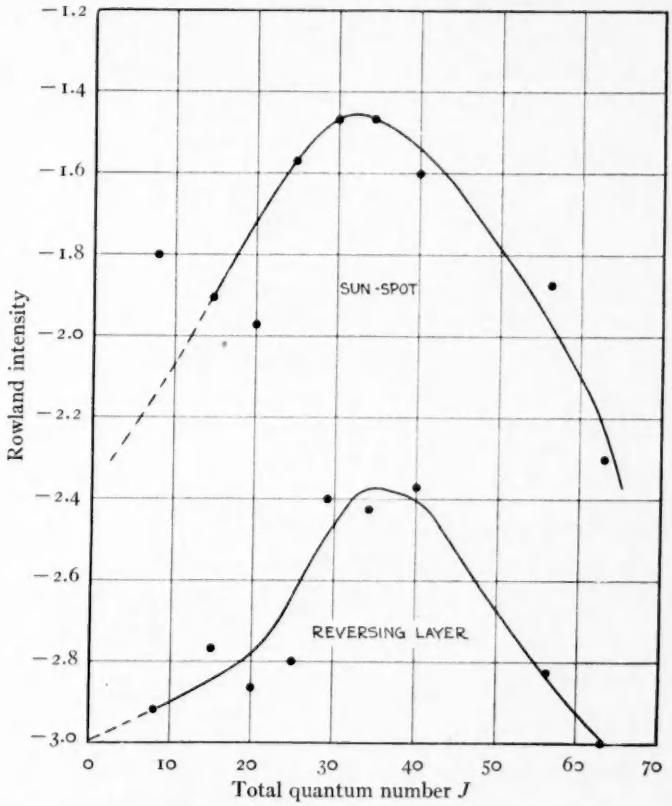

FIG. 1.—Observed intensity-curves for the R branch of the λ 5165 Swan band

reduces to $\bar{J}$, and the final working equation for the calculation of the relative intensities becomes

$$I=\bar{J}\exp.\left[\frac{-J''^2h^2}{8\pi^2I''kT}\right]. \qquad (6)$$

J. D. Shea's[1] value of the moment of inertia was used, namely, $I''=17.03\times10^{-40}$.

[1] *Physical Review*, **30**, 825, 1927.

Three methods for determining the temperature of sun-spots and the reversing layer were applied. These methods are not fundamentally different, but are simply three different ways of using (6) in connection with the observational data.

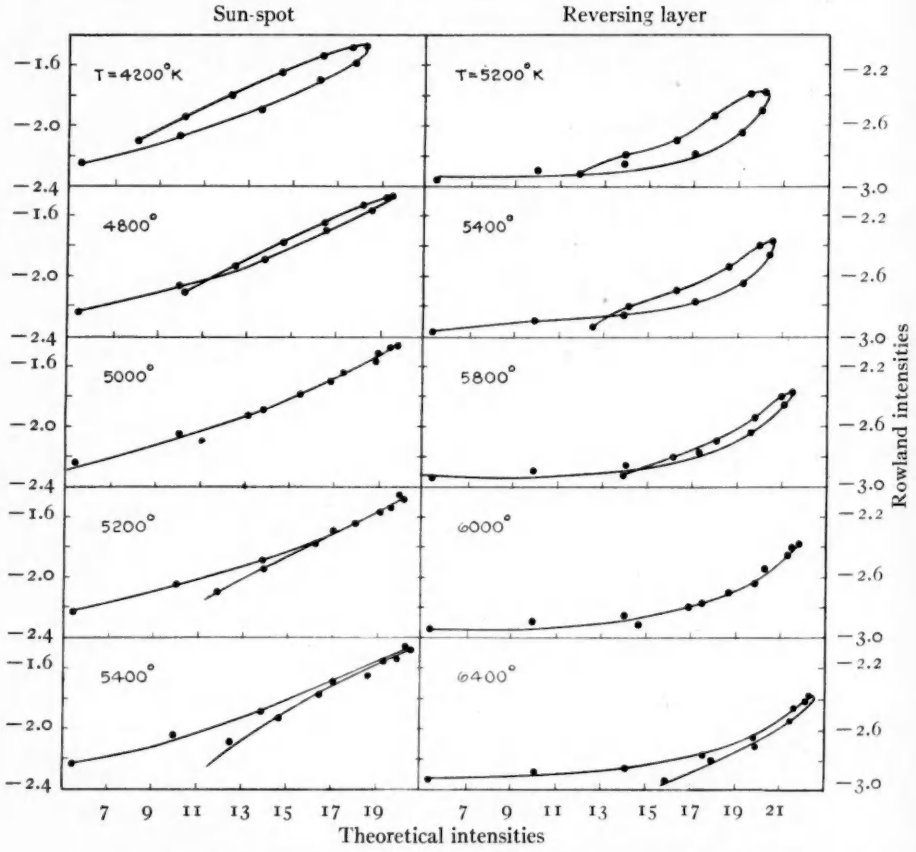

FIG. 2.—Temperature determination from comparison of observed and theoretical intensities in the R branch of the λ 5165 Swan band.

Method I.—Theoretical intensities were computed from (6) for temperatures ranging from 3800° to 6400° K and plotted against the corresponding observed intensities read from the curves in Figure 1. The resulting curves are shown in Figure 2. Most of these are double valued, since for the temperatures concerned the theoretical in-

tensities do not correspond to the same observed intensities on the ascending and descending portions of the curves in Figure 1. But obviously the intensity observed for any band line can have only one theoretical intensity, and vice versa. The curves in Figure 2 show that this condition is satisfied for a temperature of about 4900° K in the sun-spot and 6000° K in the reversing layer.

Method II.—By differentiating equation (6) with respect to $\bar{J}$, equating to zero, and solving for T,

$$T=\frac{\bar{J}^2_{\max}h^2}{4\pi^2 I'' k}, \tag{7}$$

which gives the temperatures. In this expression J'' has been replaced by $\bar{J}$, which is possible since the ratio of J' to J'' is sensibly unity. The values of $J_{\max}$ obtained from Figure 1 are 32.5 for the sun-spot and 35.0 for the reversing layer. The corresponding temperatures are T (spot) = 4900° K; T (reversing layer) = 5700° K.

Method III.—This is the method commonly used in laboratory investigations in which the true relative intensities of the band lines can easily be measured by some photometric process. The fundamental equation is derived by dividing (6) through by $\bar{J}$ and taking the logarithms of both sides. This gives the relation

$$\log \frac{I}{\bar{J}}=\frac{-\bar{J}^2h^2\,\mathrm{Mod}}{8\pi^2 I'' k}\cdot\frac{1}{T}. \tag{8}$$

I is measured for each value of $\bar{J}$, and then if $\log I/\bar{J}$ is plotted against $-J^2h^2\ \mathrm{Mod}/8\pi^2 I''k$, the resulting curve should be a straight line with slope proportional to $1/T$.

The Rowland intensities, read from the curves in Figure 1, were calibrated by the method of Russell, Adams, and Moore,[1] in terms of the relative numbers of atoms in the solar atmosphere which are active in the production of different lines in the same spectral region. The log $(I/\bar{J})$ curves obtained by use of the calibrated Rowland intensities are shown in Figure 3. The curves are linear over most of

[1] *Mt. Wilson Contr.*, No. 358; *Astrophysical Journal*, **68**, 1, 1928.

the range, and the small deviations from the straight line are probably due to errors in the observed intensities and the inaccuracy of the calibration constants for these very low Rowland intensities. The temperatures calculated from the slopes of the curves in Figure 3 are somewhat lower than those obtained from methods I and II. Method III gives $T(\text{spot}) = 4500°$ K and $T(\text{reversing layer}) = 5300°$ K.

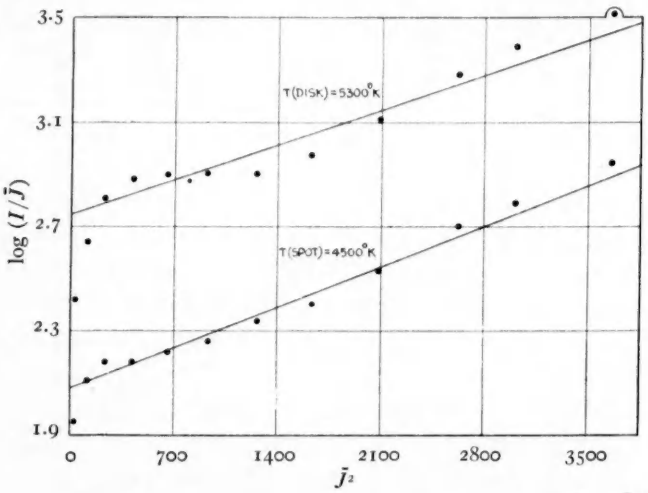

FIG. 3.—Temperature determination from slope of the curve $\log \frac{I}{\bar{J}} = \frac{-\bar{J}^2 h^2 \text{Mod}}{8\pi^2 I'' k} \cdot \frac{1}{T}$

Probable accuracy of the determinations of temperature.—The curves in Figure 2 at first sight suggest that the temperatures are fixed within rather narrow limits; but Figure 1 shows that the observed intensities are subject to considerable adjustment.

To estimate the range within which the temperatures can be determined by method I, curves were drawn through the extreme points on either side of the mean curves in Figure 1. Intensities were read from these curves and then plotted against the theoretical intensities just as before. The graphs thus obtained were much more irregular than those in Figure 2. Approximately single-valued functions correspond to sun-spot temperatures of 4400° and 5700° K, and to 5000° and 6400° K for the reversing layer. This indicates that method I gives the spot temperatures within a range of about 600°, and the reversing-layer temperature within about 700°.

In method II the error in the temperature depends entirely upon how closely J_{max} can be located. It was assumed that the range in J_{max} is represented by the horizontal distance between the two highest points in Figure 1. This is about 2.5 for the sun-spot and 5 for the reversing layer. The corresponding range in temperature is: sun-spot, 750°; reversing layer, 1600°.

If the lowest point in each of the curves in Figure 3 be disregarded, the slopes are subject to only slight variations. The range in temperature for both sun-spot and reversing layer in method III is placed at about 400°.

Comparison with temperatures obtained by other methods.—The spectral energy-curves of sun-spots have recently been investigated by Nicholson and Pettit[1] in the region λ 0.4 μ and λ 2.2 μ with a thermocouple attached to a monochromater mounted in the focal plane of the 150-foot tower telescope. The temperatures resulting from their work are compared below with those obtained in this paper. Nicholson and Pettit do not give the probable errors of their temperatures.

Russell[2] has recently concluded from a study of the electron pressure and the abundance of elements in the sun's atmosphere that the temperature adopted for the reversing layer (5040° K) is too low and should be increased to about 5600° K.

The temperatures of sun-spots and the reversing layer found by methods I, II, and III are compared in the table below with those obtained by these other investigators.

Temperature of the sun-spot umbra:	
From spectral distribution of energy (Nicholson and Pettit)	4860° K
From spectral distribution of energy corrected for scattered light (Nicholson and Pettit)	4750
From water-cell absorption (Nicholson and Pettit)	4720
From comparison of observed and theoretical intensities in λ 5165 Swan band (method I)	4900±600
From position of maximum intensity in λ 5165 Swan band (method II)	4900±750
From slope of (log $I/\bar{I}$:$\bar{J}^2$) curve (method III)	4500±400

[1] *Mt. Wilson Contr.*, No. 397; *Astrophysical Journal*, **71**, 153, 1930.

[2] *Mt. Wilson Contr.*, No. 383; *Astrophysical Journal*, **70**, 271, 1929.

Temperature of reversing layer:

From ionization theory (Russell)	5600° K
From comparison of observed and theoretical intensities in λ 5165 Swan band (method I)	6000 ± 700
From position of maximum intensity in λ 5165 Swan band (method II)	5700 ± 1600
From slope of (log $I/\bar{J}:\bar{J}^2$) curve (method III)	5300 ± 400

Although the temperatures of sun-spots and the reversing layer found by the band theory are in good agreement with those obtained by widely different methods, the large probable errors suggest that the close accordance is accidental. The methods and the observational material used in this paper are probably sufficient, however, to yield temperatures that are at least approximately correct. But most of the lines in the λ 5165 Swan band are so weak and badly blended in the sun-spot spectrum and the intensity range due to temperature is so small that it is impossible to ascertain the intensity distribution in the band with high precision. Microphotometer tracings of the λ 5165 Swan band in the spot spectrum were made in the region of maximum intensity, but so much trouble was experienced in identifying these very faint lines in the tracings, and in measuring their intensities after they were identified, that this method was finally abandoned.

Determination of stellar temperatures from distribution of intensity in band spectra.—Theoretically, measurements of the position of maximum intensity in band spectra should be an excellent means of determining stellar temperatures.[1] In deriving temperatures from the ionization theory it is always necessary to assume values for the partial electron pressure in the star's atmosphere. Further, the line intensities in different stars must be compared, which necessarily introduces many uncertainties. But the relative intensities of band lines are not affected by pressure, and the position of maximum intensity is ascertained simply by comparing successive lines in a small region of the star's spectrum.

An attempt was made to locate the position of maximum intensity by inspecting some high-dispersion spectrograms of late-type stars taken at the coudé focus of the 100-inch Hooker telescope. It was

[1] Eddington, *Internal Constitutions of the Stars*, p. 351, 1926.

found, however, that the crowding together of the lines near the band head and the presence of atomic lines completely obscure the slight rise and fall of intensity due to temperature.

The writer wishes to express his sincere thanks to Dr. Shane, Dr. Birge, and Dr. Moore for helpful advice and criticism at all times; and to Dr. Nicholson for numerous suggestions concerning the measurement and reduction of the observations. The writer is also under obligation to Dr. St. John and Dr. Russell for their interest in connection with the identification of the band lines, especially the investigation of the *SiF* bands. He is indebted to Director Adams for use of the solar equipment and for unpublished data.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
January 1931

ATOMIC SYNTHESIS AND STELLAR ENERGY. I

By R. D'E. ATKINSON

ABSTRACT

A *synthesis theory of stellar energy and of the origin of the elements* is developed, in which the various chemical elements are built up step by step from lighter ones in stellar interiors, by the successive incorporation of protons and electrons one at a time. The essential feature is that *helium*, which cannot well be formed in this way, is supposed to be *produced entirely indirectly*, by the spontaneous *disintegration* of unstable nuclei which must first themselves be formed.

A formula for the *probability of penetration of nuclei by protons* is derived from the *wave-mechanics;* it is a correction of one previously given, but agrees with that in showing that with any approach at all to the observed absolute amounts of the lighter elements *very high temperatures in normal stellar interiors are impossible.* The *model* must be approximately that *of Eddington*, but with Jeans's modification to take account of Kramers' formula for the absorption coefficient. Russell's value for the hydrogen content reconciles this formula with astrophysical data.

The low temperature and the relative amounts of the heavier elements make a *second synthesis process for protons* unavoidable; this is assumed to exist, and to possess arbitrary but extremely simple properties with no special features at any particular element. The possibility of *incorporation of electrons*, when "room" has become available, is taken for granted.

The elements whose *disintegration* supplies helium are in all cases either *those known* (or strongly suspected) to be unstable, or those isotopes, unknown now, *whose instability follows from Gamow's theory* of mass defects.

The *rate of formation of helium* must be nearly *constant*, in any one star, throughout most of its lifetime. The law of mass action demands then that all stars, after a marked *initial contraction and rebound*, should spend the greater part of their lives very *slowly expanding*. The constancy of the helium supply can be guaranteed, in the main sequence, if the average life of oxygen, until further synthesis, is about equal to the past lifetime of the star. This leads to *central temperatures* which can be calculated for the sun and estimated for other stars. The figure for the sun is *16,000,000°*, which is in *good agreement with* that calculated for *a polytrope* of index about 3 and constitution somewhat over 50 per cent hydrogen. The increase necessary for heavier stars can be covered by a small systematic change in the polytropic index. The main sequence is thus accounted for.

The *relative proportions of the elements* in stars of the main sequence follow from the theory, in *excellent qualitative agreement* with Russell's figures for the sun. The scarcity of the lightest elements, the principal maximum at a fairly early point, a minimum before the iron group, a maximum in it, a scarcity of all elements above it, and minor maxima in the barium and lead regions all follow (Fig. 2) without any special assumptions, from Gamow's theory of nuclear stability, owing to the peculiarities of the Aston mass-defect curve.

For the *low-density giants* some *earlier source of helium* must be operative. This is taken to be Be^8, whose instability was already assumed by F. G. Houtermans and the writer, and has since acquired almost the status of observational fact. It must be long-lived, since it is found on the earth, and this *accounts for the Hertzsprung gap* and its continuation between the Cepheids and B stars, and for the fact that Be^8 cannot supply helium in the main sequence. In giants it can supply enough helium, and even the brightest red supergiants can exist on the *He–Li* synthesis given by the wave-mechanics alone, with *central temperatures as low as 4,000,000°*. This figure can be reached on Eddington's theory by a systematic change in the polytropic index which is considerable but not prohibitive.

All *very diffuse giants* should be *variable*. The occurrence of *R- and N-type stars* among giants is accounted for, and also their absence among dwarfs. For the *Cepheids* an *explanation of Eddington's* has become plausible, owing to the adoption of the varying polytropic index. There should be *more very bright G and M stars* than K ones, and the *concentration of the M stars round magnitudes −5 and 0* can be accounted for. The occurrence of *coeval pairs* in which the *brighter is the cooler* is in entire agreement with the theory, as is the fact that the brightest stars in *clusters* are usually *all red or all blue*. The fact that *stars of small mass are all on the main sequence* (unless they are white dwarfs) can be explained. Various minor observations also seem to fit the theory.

White dwarfs follow from a *theory of Rosseland;* electrostatic forces will drive the hydrogen from the center of a star if it is not very abundant. They thus represent the final stage. The absence of hydrogen prevents generation of energy and they contract to a high density and temperature. Their energy may be purely gravitational, but need not be so. The *faint O stars* in planetary nebulae should all be "white dwarfs" generically.

The synthesis process predicted by the wave-mechanics is very sensitive to temperature; it is shown that *main-sequence stars* will nevertheless probably not be *"overstable."*

A number of *arguments for and against older theories* are discussed. Jeans's theory of the eccentricities of binaries need not demand a long time-scale if the galaxy is expanding as fast as the universe in general is.

A *new notation* is introduced for rapid approximate calculations. It is convenient to use and cheap to print.

1. INTRODUCTION

While the belief has now been current for more than a decade[1] that some form either of transmutation or of annihilation of matter must occur in stars, in order to provide the energy which they continually radiate, there has been little attempt as yet to investigate in detail what processes might actually occur. Treatments from the astrophysical side have in some cases made broad assumptions concerning the dependence of the rate of generation of energy on temperature and pressure, but beyond this point have generally been compelled either to introduce a large amount of quite arbitrary hypothesis or to remain so indefinite that either transmutation or annihilation could be supposed to be the fundamental process behind their equations. Direct treatment from the more purely physical side has not, until comparatively recently, been possible at all, since there was no theory of either disintegration, synthesis, or destruction of matter to start from.

During the last few years, however, our knowledge of the proper-

[1] The idea seems first to have been suggested by Jeans. A summary of the conditions that subatomic sources must satisfy was given by H. N. Russell, *Publications of the Astronomical Society of the Pacific*, **31**, 205, 1919. Cf. also Eddington, *The Internal Constitution of the Stars*, chap. xi, 1926; Russell, Dugan, and Stewart, *Astronomy*, chap. xxvi and pp. 589–92, 1927; Jeans, *Monthly Notices of the Royal Astronomical Society*, **87**, 36 and 400, 1926, and *Nature* (Suppl.), **118**, 29, 1926.

ties of atomic nuclei has advanced to a point where an a priori attack on this fundamental question can be contemplated, at least if we confine our attention to the process of atomic synthesis. In addition, our knowledge of the physical constitution of the stars, and especially of the relative amounts of the various chemical elements present in them, has advanced greatly; in a theory in which the elements are formed by synthesis this is also obviously information which can be utilized. In fact, we have now so many data for our problem that it is difficult to construct any further arbitrary hypothesis without producing a contradiction; we have some grounds for believing that if the source of stellar energy is not atomic synthesis the investigation will itself demonstrate that it is on the wrong lines. However that may be, it is possible now to work out some of the fundamental principles on which, as far as present physical knowledge indicates, probably any theory of synthesis must proceed; this is more than can yet be done if disintegration or destruction of matter are assumed to be the processes really operative.

We still know almost nothing of the conditions under which the total annihilation of matter might occur, or whether it ever could occur at all;[1] and even if we did know this, unless we knew also the laws governing its creation we should not be able to predict the relative amounts in which different chemical elements should be found in the universe. Again, we cannot yet predict the nuclear properties of atoms of atomic number greater than 92, and cannot tell for example whether, if they did exist, they would disintegrate in such a way as to provide both for the high development of energy per gram per second and for the long continuance of this development, necessary for astrophysical purposes. And here also we should be unable to make any use of such data as we possess on the relative amounts of the ordinary elements. Thus an investigation along either of these lines is forced to start by the introduction of a considerable amount of arbitrary assumption, and to continue by neglecting an appreciable mass of what might have been useful data.

When, on the other hand, we turn to processes of synthesis we

[1] Dirac, *Proceedings of the Royal Society*, **126**, 360, 1930; *Proceedings of the Cambridge Philosophical Society*, **26**, 361, 1930; Oppenheimer, *Physical Review*, **35**, 939, 1930. Cf. also Schrödinger, *Sitzungsberichte der preussischen Akademie*, **3**, 63, 1931.

have both a use for this latter material and also a certain amount of theoretical guidance as to the kind of syntheses that might occur, the probability of their occurrence, and the stability or instability of the atoms that would result. Moreover, a synthesis chain of the type we shall be studying must in any case obey certain quite definite rules. If we find in fact that we can make use of this guidance without landing in an obvious *impasse*, although its employment involves a very great limitation on the amount of arbitrary hypothesis that can still be made, there is then somewhat more justification for attempting to enlarge the theory by the adoption, if necessary, of some assumption or assumptions that may still be allowable, even if their nature is for the present purely arbitrary. The result must be judged by its success; such a theory would be required to predict, on the one hand, the relative proportions of the elements, in agreement with the estimates already mentioned, and, on the other hand, to predict also the various types of star which should be capable of a long-continued existence and therefore be prominent statistically.

Both tasks are attempted in the present paper. No effort is made, however, to achieve the first object in detail; the fact, for example, that elements of odd atomic number are rarer than those of even does not follow from the theory as it stands. It can at first sight be readily accounted for by the introduction of one assumption, whose place and nature will be obvious; but further investigation shows that the situation is a little more complex than it appears, and it has been thought better to leave details of this sort, and to make only such assumptions as are necessary on more grounds than one, even though this confines attention to the broad features of the relation between abundance and atomic number. The second object, to account for the common types of stars, has perhaps been fairly well achieved as far as qualitative agreement goes; there remain, however, some quantitative questions which demand more detailed analysis, possibly involving numerical integration of the type that has already been employed in various other investigations, before a definite pronouncement can be made. There are also certain difficulties, mostly of a minor character, that will probably demand additional assumptions for their solution. If the theory in its broad outlines should withstand criticism, it is possible, though of course

not certain, that comparatively minor alterations would strengthen it very considerably. For the present, the attempt has been made to rely as far as possible on assumptions which already have some physical support; arbitrary hypotheses are avoided until it becomes clear that even the broadest study is held up for lack of one.

2. THE THEORY OF REGENERATIVE SYNTHESIS

It is well known that the maximum possible life of the universe is much shorter if stellar energy is due to transmutation of the elements than if it is due to total annihilation. In fact, it is definitely too short to fit some theories of stellar dynamics, associated mainly with the name of Jeans, which, to say the least, have never been disproved. We shall return to this question later.[1] For the majority of other views, synthesis, provided hydrogen is consumed in it, furnishes an adequate, if scarcely a very liberal, time-scale;[2] if the element consumed is not hydrogen, the scale will be shortened from six to ten times at the least, and much more in most cases, since all other possible transformations upward or downward from known elements involve so much less percentage change of mass.

Russell has recently shown that the percentage of hydrogen in stars is probably very much greater even at the present time than had generally been supposed; in the sun's atmosphere, for example, sixty out of every sixty-five atoms are hydrogen. Since in addition the hydrogen nucleus is probably much simpler than any other, it seems very reasonable to assume that in its initial state any star, or indeed the entire universe, was composed solely of hydrogen; the small amount of angular momentum possessed by individual stars indicates that if this hydrogen was originally diffuse there was probably also very little small-scale motion in it; and the assumption that it was also cold is, though unimportant, at least as attractive as any other. The initial state of the universe thus becomes one of very remarkable simplicity, and we hope to show that the present complexity both of stars and of chemical elements could develop from this state by a self-regulating process.

At the outset there is one obvious difficulty that has been noticed by nearly every student of the question; the simplest direct synthesis

[1] §8, A, i. [2] Eddington, *op. cit.*, p. 293.

from hydrogen to any other element is the formation of helium according to the reaction $4H^+ + 2e = He^{++} + h\nu$, and this is almost certainly so improbable a process, and depends in any case so extremely on the density, that we cannot regard it as playing an important part in supplying stellar energy at all. For the same reasons a direct synthesis of any other element is even more objectionable.

If, however, helium can be supplied otherwise, progressive synthesis from this element onward is much less difficult to imagine. There is an element of every mass from 6 to 37 at least (and apparently no isobars, or duplicates, in this range);[1] and we may suppose protons can penetrate light nuclei and remain inside, and that approximately every other time this happens the nucleus becomes able to take up, and sooner or later does take up, a free electron as well. Not even three-body collisions seem to be necessary; and the theory of the penetration of nuclei by protons has in fact already been investigated and found suitable in some cases.[2]

Granted the helium then, it is not difficult to suppose that many or all other known elements (and perhaps many unstable ones unknown to us in addition) might be formed by cumulative syntheses. But if unstable elements are formed (and we know that some exist), a source of helium is provided; the process becomes completely self-supplying. In fact, if the temperature and density are suitable for syntheses in general, a rapidly increasing amount of synthesis will actually start as soon as even one other atom (stable or unstable) is present in the hydrogen. For if synthesis can build this up to a point of instability, it will presumably build up the resulting helium nucleus to the same point, and so on; each nucleus, once it reaches this point, will be able to emit α-particles—and not simply once but indefinitely. The nucleus acts as a sort of trap and cooking-pot[3] combined, catching four protons and two electrons in such sequence and at such intervals as may prove practicable, fettering them by emitting as radiation most of the surplus mass brought in

[1] On the absence of C^{14} see Menzel, *Publications of the Astronomical Society of the Pacific*, **42**, 34, 1930.

[2] Atkinson and Houtermans, *Zeitschrift für Physik*, **54**, 656, 1929. The idea of an indirect production of helium was also put forward in this paper.

[3] Cf. Eddington, *op. cit.*, p. 301.

(so as still to obey the whole-number rule at each stage), combining its captives into an α-particle, and emitting this after a delay given, on the average, by its particular decay constant. It is then ready to repeat the process, and presumably will continue repeating it as fast as external conditions allow.

The most fundamental physical objection to synthesis as a source of stellar energy is thus removed, and we may now consider in detail whether such processes of synthesis as are known or can plausibly be imagined, working in conjunction with such helium sources as are known or theoretically to be expected, can actually meet the chief requirements of the problem. The question how any process of energy generation dependent, as this one will be, on temperature and pressure can avoid leading to catastrophic "overstability" will be treated later;[1] a process independent of external conditions would almost certainly result in stars which did not fit the mass-luminosity relation and therefore were also unstable, and this difficulty seems much the more serious to cope with.

3. SYNTHESIS PROCESSES

A. THE GAMOW PROCESS: RELATIVE AMOUNTS OF THE LIGHTER ELEMENTS

We will consider first the synthesis processes that may be operative. The theory of the penetration of nuclei by α-particles was first given by Gamow,[2] and his formula for the probability of penetration can easily be modified so as to make it applicable to any other positively charged particles, in particular to protons. For small energy values of the impinging particles the formula is practically independent of the law of force in the interior of the nucleus struck; this will almost certainly be true when Z, its atomic number, is large, and may be assumed generally for the present. The probability falls off very rapidly however with decreasing velocity of the particles, and with increasing charge on either the nucleus or the impinging particle. The question whether the mere temperature movements of protons and other nuclei in stellar interiors would be violent enough for an appreciable number of nuclei to be penetrated was investigated on this basis by F. G. Houtermans and the writer;[3]

[1] §7, A; cf. also §6, C, v.

[2] *Zeitschrift für Physik*, **52**, 510, 1928.

[3] *Loc. cit.*

it was found that for small values of Z, and when the impinging particles were protons, this was actually so, but in no other case. The outlines of the theory are repeated here, although in fact it contains a rather serious error; this is corrected in a later section.[1]

The dependence on the velocity of the protons is so great that it is not the protons having nearly the average agitational energy corresponding to the temperatures in stellar interiors (assumed to be about 40,000,000°), but the exceptional protons having ten or twenty times this energy, which, in spite of their rareness, most frequently succeed in penetrating other nuclei. This obviously makes the probability of penetration per gram per second very extremely sensitive to temperature. The formula obtained by combining the Gamow formula for penetration probability as a function of velocity with the Maxwell formula for the distribution of velocities was

$$W \approx 2\xi^3 e^{-3\xi^2}, \tag{1}$$

where W is the average probability per nuclear collision (all collisions, including the slowest, being counted) that penetration will occur. ξ is the ratio of that velocity at which penetrations do most frequently occur, to the most probable velocity $v_0 = \sqrt{(2kT/m)}$ of the Maxwell formula for the distribution at the temperature T; its value is given, for protons and nuclei of charge Ze, by

$$\xi^3 = \frac{2\pi^2 Ze^2}{hv_0} \approx \frac{53{,}400Z}{\sqrt{T}}, \tag{2}$$

[1] §6, C, vi. As this paper goes to press, a recalculation by A. H. Wilson is noted (*Monthly Notices of the Royal Astronomical Society*, **91**, 283, 1931) which takes account of the radiation emitted in such processes. His formulae differ from those used here, and are probably more correct in principle, though a number of very rough numerical assumptions are involved. They retain the all-important exponential factor of the present and the earlier paper, and no formula which does this can lead to very seriously different temperatures; the actual values with Wilson's formula (with some changes in his numerical assumptions) seem to lie between those of Atkinson and Houtermans and those of this paper, but somewhat nearer the former. This would involve a systematic increase in the temperatures calculated, which might amount to 50 per cent, but can scarcely be more. Wilson's statement that synthesis has only a negligible probability is difficult to maintain in face of this, and the present theory is unaffected in its general outline.

and in all practically interesting cases it lies between $3\frac{1}{4}$ and $4\frac{3}{4}$, so that $\log_{10} W$ lies between say -12 and -28.

To obtain the actual "synthesis constant,"[1] λ, of an element of atomic number Z at a temperature T, we must multiply W by the total number of proton collisions suffered per second by a nucleus of this element, which is $N\pi\sigma^2\bar{v}$ if N is the number of protons/cc and $\bar{v}$ their mean velocity. We have thus to make an assumption about the proton density, and also about the effective "radius," σ, of the nucleus in question. The second factor is not very well known; since, however, $3\xi^2$ ranges in practice from about 30 to 70, small uncertainties in the multiplier of W play obviously a very minor rôle. In the paper referred to the nuclear radius was put at $10^{-12.4}$ cm, the temperature at $10^{7.6}$, and the density at 10^{23} protons/cc; on this basis the formula gave average lives (until further synthesis) varying from 8 seconds for *He* up to 10^9 years for *Ne*. Since the atomic number appears in the exponent in (1), the average life increases very rapidly with each step up the periodic system.

The dependence on temperature is also very marked; thus logarithmic differentiation of (1) and (2) shows that W varies approximately as $T^{2\xi^2}$; the exponent is normally between 16 and 22, being largest when Z is large and T small, but it may fall as low as 7 for the lightest elements at the highest temperatures considered.

Roughly speaking, for every proton which enters and remains within a nucleus, energy corresponding to .007 units of atomic weight will be liberated. This number varies a little according as the mass defects of the element formed and that destroyed differ in one direction or the other; broadly speaking, however, the amount of energy is always nearly the same. Thus it is possible to calculate the numbers of atoms of the various sorts that should be present in a star if the observed energy development is to be accounted for entirely by their conversion to heavier elements. The order of magnitude will be given if we take one kind of atom only, and assume that as many atoms are being produced per second as are being destroyed, and that these syntheses liberate the observed amount of energy; if the

[1] This is the exact analogue of the "decay constant" in radioactivity, and its reciprocal, τ, will often be called the "average life" in what follows, where there is no danger of confusion.

average life of the atoms is long, there will have to be many of them, and conversely. The calculation was made for the case of boron, on the assumption that the temperature was sufficiently high only inside a sphere of radius $10^{9.8}$ cm, and that 10^{35} erg/sec. were to be accounted for (case of a large main-sequence star); a density of B corresponding to $10^{17.7}$ atoms/cc was obtained, which seemed quite a reasonable figure.

The choice of boron for the comparison was partly fortuitous, and the correctness of the figure obtained was at that time purely conjectural; it is, however, now possible to make a considerably more extended and more satisfactory check. Clearly if all the (lighter) elements really are subject to this synthesis process, and if known isotopes are to result from it, we are immediately compelled to allow the assumption that the newly formed nuclei can also sometimes take up electrons. For example, the result of incorporating a proton in a boron nucleus of mass 10 would be a carbon nucleus of mass 11; this does not exist on the earth, but B^{11} does, so that presumably a C^{11} nucleus has room for one electron more, and takes it up in a time which is short compared with 10^9 years. Since nuclei do not, as far as we know, actually repel electrons at all, this process need not perhaps require a high temperature; even if the C^{11} atom happened to reach a cold region after being formed, and to remain there, we should not necessarily expect to find it capable of continued existence.

If we accept this view, we can read off at once from the list of isotopes the points at which another electron does get taken up; we cannot read off the probability of the process, but we can at least say that where isobars exist the probability must be very small. Among the lighter elements this is apparently never the case, and we shall assume provisionally that here the probability is in fact quite considerable. We shall also assume that whenever a proton penetrates a nucleus it remains inside, so that penetration and synthesis become synonymous terms; this is almost certainly not quite correct, though it may perhaps hold when the resulting Z is even (or alternatively, when the resulting atomic weight is divisible by 4); in other cases, we should perhaps assume a lower probability of the proton's remaining "anchored," but it is, as already men-

tioned, preferable to disregard refinements of this nature for the present.

With these preliminaries, and still supposing that the supply of helium is somehow maintained constant at a suitable rate, we can predict the relative amounts of all the lighter elements in a star; we can also predict the absolute amounts from the total energy generation if we know how much hydrogen the star contains, but since this is somewhat uncertain, the relative amounts provide the more reliable check. We have, in fact, an analogous case to the well-known law of radioactivity, that the amounts of the various members of a series in the state of equilibrium are directly proportional to their average lives; the fact that in our case the series is a synthesis series and not a disintegration series is quite immaterial if only the *He* supply is maintained constant. Accordingly, the average lives, $\tau = 1/\lambda$, calculated from (1), should be directly proportional to the amounts of these elements present.

Recent work of H. N. Russell's[1] gives us much better information about the amounts of the various elements in the sun's atmosphere than we had previously. It is not of course certain that these amounts are a fair indication of the relative amounts in the whole body of the sun, but if mixing is strong enough to outweigh the principal selective factors this will be true. Differences of atomic weight will tend to concentrate the heavier elements toward the center of a star, but this, as Eddington has shown,[2] occurs only at a negligible rate in a normal star. Differences in the power of absorbing radiation may drive some elements outward, but different elements will be affected most in different parts of a star, and in any case the radiation-pressure gradient is small in the sun compared with the gas-pressure gradient. Differences in the value of e/m will drive some elements outward if there is a radial electrostatic field, as there probably is, but e/m for fully stripped nuclei is practically constant from $Z = 2$ to $Z = 20$ so that such differences will occur mainly in the outer layers as far as the lightest elements are concerned. The effect may be appreciable for hydrogen, and perhaps for helium; we shall return to this point later.[3] Helium is in any case

[1] *Astrophysical Journal*, **70**, 11, 1929.

[2] *Op. cit.*, p. 279. [3] §§ 6, D; 7, C.

in an exceptional position. On the other side we have the visible evidence of sun-spots, and also of differential rotation, to show that there really is a very considerable amount of large-scale motion in at least the outer layers of the sun; the polarities of sun-spot pairs, and the large total amount of matter thrown up in the more permanent spots, seem to indicate that a sun-spot is in fact a fairly deep-seated phenomenon. And it will appear below that, at least in the main sequence, an appreciable amount of stirring is probable at all great depths also.[1]

Provisionally it seems reasonable then to assume that mixing is roughly powerful enough to guarantee that Russell's figures indicate the relative amounts of the elements in the whole sun. We may thus compare the values of log τ given by Atkinson and Houtermans with the values of log T (T being proportional to the number of atoms of an element) given by Russell. Table I gives values of log N (where N is proportional to Russell's T) obtained by combining his Tables XIV and XX; below them are put the values of log τ taken directly from Atkinson and Houtermans. Both sets of figures run over a very considerable range and rise in a roughly similar manner; considering the admitted uncertainty of Russell's figures for these elements and the very rough nature of the theory, we may perhaps fairly speak of a good qualitative agreement here. (The more complete theory we shall develop involves several very important corrections to these figures; the final agreement is improved.) The striking scarcity of *Li*, *Be*, and perhaps *B*, in a region of otherwise plentiful elements, noted by Russell as one of the most prominent of the broad features of his results, appears in this theory as a direct consequence of a very natural application of the wave-mechanics; these elements are too easily destroyed to become plentiful. It was this fact which first suggested to the writer that an at-

TABLE I

	Li	*Be*	*B*	*C*	*N*	*O*
log N	5.0:	4.8	8.5::	10.4:	10.6?	11.4
log τ	3.3	5.5	7.6	9.5	11.4	13.2

[1] § 7, C.

tempt to modify the theory, so as to make it applicable to a much wider range of phenomena, was perhaps worth serious effort.

B. CENTRAL TEMPERATURES: MILNE'S THEORY

Above oxygen there is no longer any sort of agreement between the lives τ and the amounts of the elements. The calculated τ-values get larger and larger, and very high temperatures indeed would be necessary if they were still to remain comparable with 10^{10} years at large Z-values. This difficulty was recognized by Atkinson and Houtermans; there seems no escape from the conclusion that in fact some other synthesis process, not dependent on Z in this very marked way, must also exist. In particular, it is difficult to suppose that the high temperatures really are there, because the relative and absolute amounts of the elements of medium and large Z are quite unsuitable. The proportions should continue to rise until that element was reached whose average life was comparable with the sun's life, and after that they should fall rapidly. But the maximum is actually reached at oxygen, the only rapid fall is after *Zn* and does not continue indefinitely, there is a well-marked second maximum before it, and the absolute amounts of the elements up to oxygen are already so large that a very great deal too much synthesis would occur even at some distance from the sun's center if there were any very high temperatures. The absolute amounts are indeed somewhat uncertain, since it is possible that there is relatively more hydrogen in the outer layers of the sun than elsewhere; a calculation carried out on the same lines as that of Atkinson and Houtermans shows, however, that the amounts are roughly adequate for the sun's energy supply, if we assume such a temperature that the most abundant element, oxygen, has a lifetime comparable with that of the sun itself. This is quite a low temperature, and any considerable increase in it would create a very difficult position.

It should perhaps be stressed at this point that the penetration of nuclei by protons at the temperatures we are considering is really a phenomenon whose occurrence it is somewhat arbitrary to deny in the present state of the wave-mechanics. The Gamow formula has scarcely been questioned in respect to its qualitative applicability (though various objections, on mathematical grounds, to some

of the ways of deriving it have been raised),[1] and it seems in fact to be qualitatively correct in the realm of artificial disintegration of the elements. If then the synthesis process here sketched does not in fact occur, it is practically necessary to assume specifically that the protons which we know get into nuclei do not succeed in staying there. Provisionally it seems more reasonable to suppose that a proton has in fact a fair chance of radiating at least $10^{-4.5}$ of its mass when it penetrates a nucleus, and so of remaining bound inside it; but, at any rate, the assumption that the proton can often stay in the nucleus is not now more arbitrary than the assumption that it practically never can.

The temperature assumed by Atkinson and Houtermans is that given by Eddington; Milne[2] has recently criticized Eddington's theory on the ground that stars with a point source of energy, and a point concentration of mass, at the center (or a reasonable physical approximation to this arrangement) would be more stable than Eddington's models; in such stars the central temperatures would be very high indeed. He claims that there is nothing in the mathematics of Eddington's analysis to rule these models out, and that in fact only the assumption of some form of dependence of energy generation on temperature can do this. Eddington appears to have originally excluded the possibility by the assumption that the point source of energy would "blow away" the point mass by reason of the intense radiation pressure gradient it would introduce,[3] and this, though it would perhaps be sufficiently true in many physical realizations of the mathematical picture,[4] is possibly not an absolutely adequate answer to Milne's objection. However that may be, our theory seems able to satisfy Milne's requirement more completely; with the kind of energy sources given by the wave-mechanics, the concentration of any appreciable fraction of a star's mass at the temperatures inferred by him would result in a development of

[1] Cf. p. 257, n. 1.

[2] *Observatory*, August, 1930, and references; *Nature*, **127**, 16, 1931. Larmor, *Observatory*, June, 1930, and references; *Monthly Notices of the Royal Astronomical Society*, **91**, 4, 1930.

[3] Eddington, *op. cit.*, p. 125.

[4] The general theory of relativity will be among the factors preventing a close approach to a "point mass" solution in any actual case.

energy sufficient to explode the star almost instantly. In point of fact even a very small concentration will increase the rate of generation very markedly, and the star will thus expand again whenever such a situation even threatens to develop. The only exception would seem to be when there is no hydrogen near the center; this case is discussed below.[1]

Thus we have an argument, from the rate of energy generation, which makes point masses difficult a priori, and this must be added to the argument already drawn from the absolute amounts of the light elements and the fact that the medium-heavy ones only decrease slowly in abundance. (It may be noted here that the qualitative similarity of the abundance curves for terrestrial and solar elements, especially as regards the maximum at iron and the even higher one at oxygen, is perhaps evidence that the sun is of roughly the same constitution throughout; oxygen, for example, cannot well be supposed extremely scarce at the center.) There is also the chance of obtaining evidence from the rotation of the periastron in binaries;[2] conclusions are not yet very certain, but concentrations of appreciable mass at the centers of stars seem already to be improbable on this ground.

If we grant the physical impossibility of point-mass solutions of the equations, we have in effect granted the essentials of Eddington's entire theory. In particular, we have then a detailed explanation of the mass-luminosity relation; and we have also the means of making at least rough estimates of the temperature and density at any point in a star. This position is therefore the one on which we shall proceed in the present inquiry; there is however reason to modify the theory at one point in a way proposed by Jeans.[3] The modification consists approximately in using polytropic models for which the index increases with the mass, instead of models with a constant index.[4] The order of magnitude of the central temperatures is not affected by this however, and it follows that we must assume also

[1] § 6, D.

[2] Russell and Dugan, *Monthly Notices of the Royal Astronomical Society*, **91**, 212, 1930.

[3] *Monthly Notices of the Royal Astronomical Society*, **87**, 36, 1926.

[4] The question how far this will affect the $M-L$ relation has apparently not been investigated thoroughly. Since the relation exists, we have assumed the question is not serious; it is hoped to study it later.

some other synthesis process, in order to reach the heavier elements at all; it will appear below that not even the assumption that, at an early stage in its history, the sun had high central temperatures can help us otherwise.

C. A SECOND SYNTHESIS PROCESS

Without further justification at present, we therefore introduce now an arbitrary *ad hoc* assumption. In addition to the simple synthesis, "process A," which we have been considering, we postulate a second process, "process B," defined by the following conditions: (1) Its probability increases with increasing Z, especially at small Z-values, but changes less rapidly with Z than that of process A; a probability increasing by a factor of 10 between *O* and *Si*, by another factor of 10 between *Si* and *Ti*, and perhaps by another factor of 100 between *Ti* and the heaviest elements, would be suitable. (2) Its absolute probability under solar conditions is comparable with that of process A in the oxygen-fluorine region, so that for all smaller Z-values process A is much the more probable, and for all larger Z-values process B. (3) Like process A, it must be very sensitive to temperature. We cannot specify any definite figure, nor do we need to; but a variation with the twentieth or thirtieth power of the temperature would apparently be by no means too large at small Z-values. It may be less if the synthesis probability (per atom) increases with less than the first power of the density; this possibility, however, it not attractive. At large Z-values we are not apparently bound to any specific conditions.

It is only with reluctance that we introduce such a hypothesis, since there is no a priori justification for it at all, as far as is known. At the same time, an arbitrary hypothesis of a very definite nature is perhaps preferable to an indefinite one; if this hypothesis cannot in fact be allowed, many apparently possible approaches are blocked. If this investigation does nothing else, it may clarify the issues by showing on what terms a synthesis theory can be worked out. Of course, it would be too much to claim that no other terms could be found; but the theory here presented is in fact the only survivor of a considerable number of alternatives that all promised well at the start, and all had to be abandoned, often at quite advanced stages.

While a probability depending very markedly on temperature

is easy to imagine, a probability increasing with increasing Z is apparently less so. It is conceivable that in three-body collisions of the type nucleus-proton-electron the proton and electron should act as a sort of temporary neutron (lifetime less than 10^{-20} sec.), and that such collisions should sometimes result in penetration on the proton's part; a probability of $10^{-8} - 10^{-12}$ per three-body collision would be adequate, and at least a comparative independence of Z might be expected. Whether the probability could actually increase with Z is more doubtful; but in any case it would have to vary with the square of the density, and this last characteristic appears, as we shall see, to be quite inadmissible. Nuclear "resonance" has repeatedly been suggested as allowing penetration very much more easily than it would occur on Gamow's theory, but this appears a somewhat slender hope.[1] It is however possible that the interaction of protons and radiation should result in some penetration; at least only the first power of the density would then be involved. However that may be, the actual justification for the assumption of process B is to be sought entirely a posteriori for the present.

If both processes are in fact operative, a study of Russell's figures indicates that we may suppose the elements up to nitrogen, and also in the *Mn-Ni* region, are present in equilibrium proportions, those in the interval in somewhat less than equilibrium proportions, and those above *Zn* in much less. This is exactly the situation we require; in order to show this, however, we must turn now to disintegration processes.

4. DISINTEGRATION PROCESSES

The criterion for the stability or instability of nuclei in general is also due to G. Gamow.[2] He gave an explanation of the general form of the mass-defect curve, and called attention to the fact that it is the sign of the slope of this curve which decides stability, not the sign of the actual ordinates. If the ordinates are positive, the atom could indeed completely disintegrate into α-particles, with liberation of energy, but they are not positive even among the radioactive elements. From about *Xe* onward, however, the slope is positive,

[1] Cf. Atkinson, *Zeitschrift für Physik*, **64**, 507, 1930, and A. H. Wilson, *loc. cit.*

[2] *Proceedings of the Royal Society*, **126**, 632, 1930.

and at any point in that range the expulsion of at least one α-particle should at first sight be possible.

The mass-defect curve of the existing elements is however an envelope of a number of constituent curves differing in respect of the number of "free" electrons (in addition to those needed for α-particles) contained in the nucleus; thus even on the upward-sloping part stable elements are possible, if they happen to be near the lowest points of one of the constituent curves, since an atom cannot easily "jump" from one curve to another; and even on the downward-sloping part unstable elements are sometimes possible among those which contain fewer free electrons than they could hold. Figure 1 shows cases in point. The places where the successive curves cut each other cannot yet be obtained exactly from theory (though for fairly light nuclei of atomic weight $4n$ Gamow's investigation gave qualitatively the right results), but they are in many instances known exactly from the list of isotopes. Thus for atomic weights $4n$ there are no free electrons in the nucleus (in addition to those needed to cement the protons into α-particles) until we reach the atomic weight 40, when the nucleus is apparently stable both with and without two free electrons (A^{40} and Ca^{40}); from then onward until 64 inclusive, there are always two; from 68 to 80, four, and so on. We are thus able to sketch in at least the stable parts of the constituent curves.

Roughly we may say that as long as the slope of the envelope curve is negative unstable elements may be expected to be somewhat uncommon; it is not enough that the nucleus should lie on a constituent curve which has left the envelope; it must have left it and turned upward, and the first (or sometimes second) element (and all later ones) past the minimum will then be unstable. As soon, however, as the envelope itself has turned upward, the possibilities for instability become more numerous.

It is not surprising that no such unstable elements are known on the earth, at least until we reach those that are continually being replenished from above. For we should expect them to have fairly short lives as a rule. The energy with which an α-particle is emitted depends on the steepness of the curve on which the atom lies, and where the envelope slopes upward the constituent curves are prob-

ably steep and the corresponding disintegration will be violent. Where the envelope slopes downward it would be possible that disintegrations of very small energy should occur; but, on the other hand, the nuclear charge is so small here that even these disintegra-

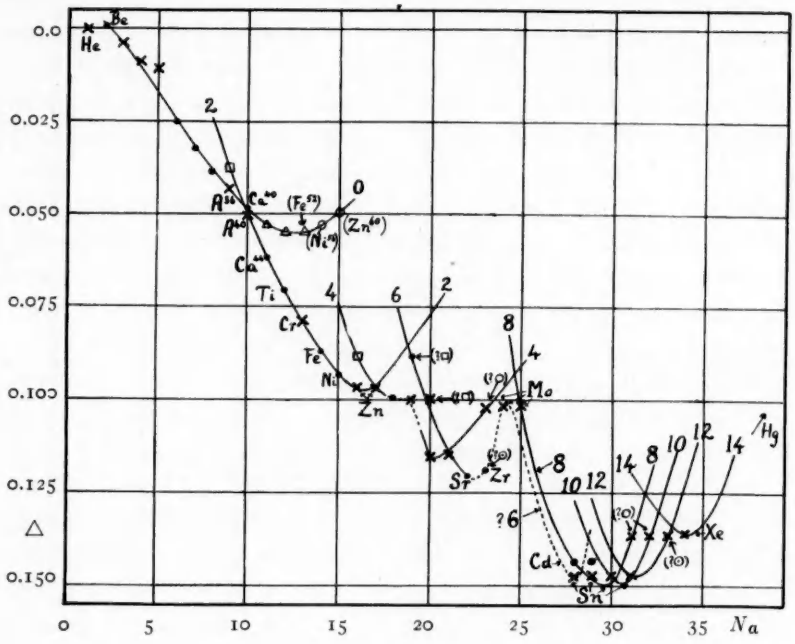

FIG. 1.—Mass-defect △ in terms of number of α-particles in nucleus. (After Gamow.)

× Known, mass-defect measured
• Known, mass-defect not measured
△ Ionized but not α-unstable
○ Ionized and α-unstable
⊙ α-unstable but not ionized
□ β-unstable
} Existence not verified.

tions would hardly be expected to have very long periods on the average. If a long-lived element should occur, the disintegration energy would necessarily be very small indeed; it should be stressed that no systematic search has yet been made for elements which are unstable, but whose disintegration liberates too little energy to cause the more usual phenomena of "radioactivity," and it is quite possible that some such cases exist.

Gamow's theory was developed actually for elements of atomic weight $4n$ only. There seems, however, no reason to expect that it should not apply equally well to all others. But there will then be the additional possibility that the nucleus should sometimes be proton-unstable; this will occur when the actual nuclear atomic weights of two successive elements differ by more than 1.007. Among the isotopes that no longer exist this may have been common, but it is comparatively irrelevant for our purpose; it would mean in effect that that particular proton-synthesis did not actually occur at all, there being in fact no stable level for the proton to fall to after penetration. In other words, if we suppose the synthesis to have occurred, the nucleus will reverse it by disintegrating again with a decay period which might be as short as 10^{-20} seconds (though of course if the energy-level were very nearly zero the period could be comparatively long). Synthesis past such a point would be dependent then on the adoption (or perhaps the emission) of one or more electrons, before the next proton could be assimilated.

In addition to possible proton-unstable elements of very short life, we actually know of α-unstable ones with decay periods as short as 10^{-5} or 10^{-6} seconds. These may play an important part, as we shall see. They will not however necessarily prove serious obstacles to further synthesis, for they happen to occur at natural branching-points in the radioactive series, and the synthesis could perhaps sometimes take the electron branch. Probably more important is the fact that the fourth (now nonexistent) radioactive series must have played as large a part as the other three in the formation of *Th* and *U*, if synthesis, as we are supposing, does not proceed in larger than proton steps.

To summarize then: α-unstable elements will probably be fairly common among those reached by any synthesis (or disintegration) process that is active above about *Xe* ($Z = 54$); we may in fact almost speak of it as an accident if elements are stable once this point has been well passed. The decay periods may possibly be fairly short in many cases, but (owing to the moderately large nuclear charge) perhaps not extremely short as a rule. Below $Z = 55$, α-unstable elements may also occur at certain points; a necessary condition is that they, and perhaps several of the elements that precede them,

should have fewer electrons than they could hold; this can only occur in a region where more or less permanent isobars are possible, but some very permanent ones are found down to $Z = 20$. In particular, we shall assume that α-instability occurs at the first point at which it could, namely, as soon as the first constituent curve has turned upward. This will apparently be in the *Fe-Zn* region, and we may for convenience in discussing details specifically suppose that the α-unstable element is Zn^{60}; the product of disintegration will then be Ni^{56}, which could become Fe^{56} again by electron captures. This definiteness is in no way vital to the theory; it is really only a shorthand notation for "the disintegration product," etc.

5. THE RELATIVE AMOUNTS OF THE ELEMENTS

A. EARLY HISTORY OF A STAR

We have now all the major physical assumptions that we shall need, and may proceed to discuss their astrophysical consequences. The introduction has been somewhat lengthy, but this is more a tribute to the novelty of the subject than to the number of the assumptions. We have in fact assumed only: I, the existence of process A, which follows from the wave-mechanics together with the assumption that a proton which penetrates a nucleus usually remains inside; II, the possibility that nuclei should take up additional electrons, often with a fairly high absolute probability, as and when the list of "stable" isotopes shows that the incorporation of protons has made fresh room available;[1] III, the existence of process B, defined by the rough absolute value of its probability and the qualitative dependence of this on Z and T; IV, the existence of disintegration at about the points indicated by Gamow's theory. This involves the subsidiary assumption that the probability of electron penetration is on the average rather less than that of proton penetration in the regions in question; the existence of isobars may be adduced in support of this. Of these four assumptions, three are sufficiently[2] a part of at least the tentative physical theories of

[1] This idea goes back at least to 1923; cf. Harkins, *Philosophical Magazine*, **42**, 305, 1923; G. Beck, *Zeitschrift für Physik*, **47**, 407, and **50**, 548, 1928. Electron probabilities are much more difficult to calculate than those for heavy particles, since they demand relativistic wave-mechanics. Cf. Beck, *ibid.*, **61**, 615, 1930.

[2] How much may be seen, for example, in Chadwick, Constable, and Pollard, *Proceedings of the Royal Society*, **130** 463. 1931

the day to make their application to astrophysics well worth trying; the remaining one, though arbitrary, is at any rate more definite than most of the arbitrary assumptions that have been tried in this field. On this basis then, and in other respects following Eddington, we hope to account roughly both for the relative abundance of all the chemical elements and for the existence of all the principal classes of stars.

It should perhaps be stressed that these assumptions, and particularly the assumption of process B, are all already used in accounting for the relative amounts of the elements; having done this, we require no further hypothesis at all in order to account for the great majority of the stars. We shall see that if process A alone existed, and the central temperatures of stars were high enough for the synthesis of an unstable element in the iron region to maintain the helium supply, a considerable amount of mixing would still be probable. In that case, no element below, say, sulphur ought to be present in detectable amounts in the sun's atmosphere at all (since τ falls off so rapidly with decreasing Z), and certainly the observed absolute amounts of many of the lighter elements would be more than 10^{10} times too large.[1]

In a star consisting of nothing but hydrogen, the synthesis processes we have been discussing could not occur. But we are not of course forced to assume absolutely pure hydrogen, and even if we do, it is conceivable that once in ten million years somewhere in the star a helium nucleus might result directly from a six-body collision. As soon as we have one nucleus, if the conditions are suitable for synthesis others will rapidly be generated as already explained; but it is clear that a certain "incubation period" will be necessary before enough synthesis could be occurring to supply the star's energy needs. The early stages of a star's history, which are in fact of some importance, will then necessarily be as follows:

At first the star will be a diffuse mass of hydrogen, of uncertain shape and temperature. As it gradually packs down into the spherical form it rises in temperature and starts radiating; its radiation will in "equilibrium" be determined by its mass, though (owing to the

[1] This argument must be modified in view of that given in § 7, *C*. It seems however unlikely that a self-consistent high-temperature theory can be constructed along such lines.

small atomic weight) it will perhaps not exactly fit the ordinary mass-luminosity curve. The equilibrium is however somewhat transient, since the energy must be supplied almost entirely by gravitational forces at first; the sun, for example, requires $10^{38.5}$ syntheses per second to meet its energy expenditure, and this number cannot be reached offhand. Nevertheless, as the temperature rises, we suppose that synthesis does begin. It will then continue at a rate which is on two grounds bound to rise increasingly fast. First, the synthesis processes will be becoming steadily more probable, owing to the rising temperature and density; and, second, the supply of helium will be rising at an increasing rate, owing both to this fact and to the rise in the number of helium-producing atoms.

The last statement may however require some modification at first. The average life of any α-unstable atom, with respect to disintegration, is fixed; its average life with respect to possible further synthesis is, on the other hand, getting shorter and shorter as the temperature rises, so that disintegration is becoming less and less probable. In general, then, it is to be expected that every disintegration process will get "smothered" sooner or later, unless it is a very violent one; unstable atoms will continue to be formed, but they will be built up further by the addition of more protons (or of electrons), before they have time to emit an α-particle. This will check, and may more or less completely cut off, the supply of helium, at least at certain stages, and contraction will have to continue until some atoms are reached that are too violently unstable for their disintegrations to be suppressed. Whether this would occur before the ordinary radioactive elements are reached we cannot say; but at least when these are reached very violent sources of α-particles will come into action, and moreover every single element formed in any one of the four series will be unstable in some way or other.

From this point onward, as long as the temperature does not decrease, the amount of synthesis per second will certainly increase rapidly and at a rising rate, and it appears fully probable that now at least the amount of energy generation should become adequate for the star's needs. Contraction will then naturally stop; a contraction at the Kelvin rate is of course slow compared with possible pulsation motions, and there will be no marked discontinuity, but

the star will gradually change over in type from one in which gravitation is supplying the energy ($\epsilon \propto T$ in Eddington's notation) to one in which subatomic processes, acting mainly near the center, are supplying it.

It will however by no means be true that an almost stable state has now been reached, but quite the contrary; the position is on the point of becoming more or less violently unstable. For the cessation of contraction will not prevent the generation of helium from going on at an increasing rate, owing to the fact that even at constant temperature, and with constant helium supply, the amount of helium-producing elements increases at a steady rate. (Exhaustion of the hydrogen is still too small to play any serious part.) The star will obviously be compelled to expand again, in order to keep the total rate of synthesis down to the figure suitable for its mass; the increased amount of helium sources will thus be countered both by decreasing the rate of synthesis in general and by decreasing in particular those syntheses which produce helium from helium sources. But the expansion will nevertheless to some extent defeat its own end; for the lengthened synthesis periods which it introduces will allow those disintegrations which had been smothered to become operative again; every effort the star makes to cut down its helium supply will actually bring new helium-producing disintegrations into action.

It is thus possible that the expansion should take a quite catastrophic form, though this is not certain. But at least the star will not gain control again until further expansion, in addition to diminishing the activity of such sources of helium as are operative, brings no new disintegrations into action. This will presumably only occur when there are no more disintegrations to be reached, or at least none very easily reached; in other words, the star will expand until Zn^{60} (or whatever it is that is unstable in this region) has time to disintegrate. This point might even be overshot if a large amount of unstable elements, or of free helium, has been formed, but, unless some still lighter radioactive element exists and becomes operative, the star will settle down into a state in which the main *He*-supply is the disintegration of Zn^{60}, and the conditions are such as to maintain an adequate rate of regeneration of this element.

B. THE ELEMENTS ABOVE ZINC

It will appear below that the expansion must probably have been quite considerable, and its effect will be to make the absolute probability of process B much smaller than at maximum compression; there will then be comparatively little further synthesis above $Z = 30$, since the absolute amounts of the elements present are still fairly small and the disintegration of Zn^{60} prevents any further supply of material.

If this is the case, while the distribution of the lighter elements is affected by the fact that their amounts are still being steadily added to, the present distribution of the heavier elements should simply represent a steady state in which a small continuous amount of synthesis is being compensated by an equal amount of disintegration, and not complicated by any appreciable feeding from below at all. This may certainly be expected to involve a maximum of abundance in the region $Z = 55$–60, since these are the elements of least potential energy. Elements that already were unstable would tend to "slip down" to this point directly. Elements that were near a minimum of one of the constituent energy-curves of Figure 1, and were therefore stable, would sooner or later pick up a proton, which would transfer them to a similar diagram for atomic weights $4n+1$, in which they might easily find themselves well above the minimum of some curve; in this way even the "stable" elements above $Z = 60$ would gradually work their way down to this region.[1] Elements already in this region would occasionally capture protons and electrons and work their way up a little, only to fall back again. Among the heaviest elements of all, however, lead would be continually supplied from above, and might then by the same sort of process keep one or two others in its neighborhood replenished; since mercury has at least seven isotopes it appears that there may be rather a pronounced "step" in the envelope of the energy-curves in this region. Thus from $Z = 55$ upward we shall expect to find a marked maximum of abundance at first and possibly a secondary maximum in the lead region. This expectation is entirely fulfilled.

[1] There appears a possibility that radiation (X-rays) may accelerate the passage of nuclei from one of the curves to another, but probably not by emission of protons. Cf. G. I. Pokrowsky, *Zeitschrift für Physik*, **63**, 561, 1930.

Figure 2 shows the amounts of the various elements in the sun's atmosphere, plotted from Russell's data. The ordinates are logarithms of the numbers of atoms (Russell's log T), not the weights

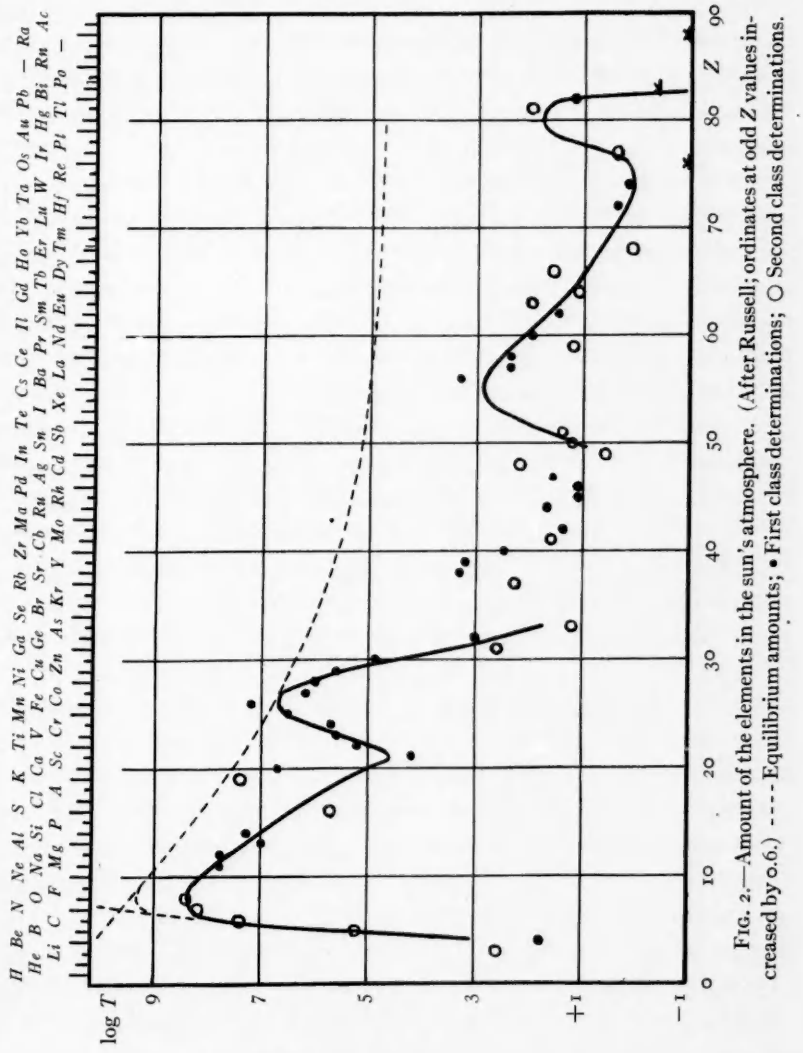
H Be N Ne Al S K Ti Mn Ni Ga Se Rb Zr Mo Pd In Te Cs Ce Il Gd Ho Yb Ta Os Au Pb — Ra
He B O Na Si Cl Ca V Fe Cu Ge Br Sr Cb Ru Ag Sn I Ba Pr Sm Tb Er Lu W Ir Hg Bi Rn Ac
Li C F Mg P A Sc Cr Co Zn As Kr Y Mo Rh Cd Sb Xe La Nd Eu Dy Tm Hf Re Pt Tl Po —
log T
9
7
5
3
+1
−1
0
10
20
30
40
50
60
70
80
90
Z

FIG. 2.—Amount of the elements in the sun's atmosphere. (After Russell; ordinates at odd Z values increased by 0.6.) ---- Equilibrium amounts; • First class determinations; ○ Second class determinations.

of the elements, so that the diagram differs somewhat from that given by Russell. We have also incorporated in it the revised estimates for oxygen and hydrogen given in his Table XX, and have

added 0.6 to the logarithms of the amounts of all elements of uneven Z. This last step was taken purely to eliminate the well-known "zigzag" feature of all such diagrams, so as to bring out the major features more clearly; in any numerical estimates based on this figure, the fact must be allowed for.

It is evident that from $Z=45$ upward the situation is qualitatively just what would be expected; the maximum among and below the lighter rare earths is quite appreciable. Immediately below $Z=45$ it is not yet possible to make definite predictions. It seems that we must wait for more accurate determinations of the mass defects, since Aston's figures[1] would involve, if correct, both a curious inflection in the curve for six free electrons (in order to bring it down to Sn^{112} at $N_\alpha=28$) and also radioactivity (α-rays) on the part of Mo^{96}, Zr^{92}, and Mo^{92}. If the mass of Mo^{96} is somewhat less, and that of Sn^{112} somewhat more, than Aston's figures (he estimated his probable error for Sn at .024, which would be nearly enough), the inflection could be wiped out; in that case there may be a long flat minimum of the curve in this region, with a corresponding tendency for elements to accumulate here, in accordance with the actual state of affairs. It is of course also possible that there is a small amount of supply from smaller Z-values in this region; iron is over ten thousand times, and nickel one thousand times, as abundant as germanium, so that if only a very small fraction of the unstable atoms succeeded in getting past the danger-point without disintegrating, the relative effect from $Z=32$ upward might be considerable. At worst, the amounts in this region present no specific difficulty.

Qualitatively then we have accounted for the main features of the "abundance-diagram" from $Z=32$ upward. It should perhaps be stressed that our really "arbitrary" assumption has nothing to do with these features individually; it was a postulate of almost constant synthesis probability through the whole region. The differences in abundance have followed from already propounded theories, apparently unescapable, about atomic stability. They will arise if there is any small amount of synthesis in this region.

[1] *Proceedings of the Royal Society*, **115**, 487, 1927; *ibid.*, **126**, 511, 1930; *ibid.*, **130**, 302, 1931, and references.

C. THE ELEMENTS FROM CARBON TO ZINC

i) *Disposal of final synthesis products.*—In addition, Russell's figures show that there is a very marked contrast in abundance between the elements from carbon to zinc and those of larger *Z;* this is such a prominent feature of his diagram that any theory of atomic synthesis must take cognizance of it, and it follows quite necessarily from our assumptions, since the former elements are the only ones that have continued to be created and left in existence throughout most of the sun's history. The unstable element that during this time has steadily blocked the synthesis of higher elements must clearly be assumed to be at the point where the rapid fall in abundance occurs, and this is exactly[1] where Gamow's theory had already placed it.

We turn now to these lighter elements, from carbon upward, whose formation has been responsible for the sun's energy development throughout most of its history. At the outset we may note that the element most abundant of all in the sun must have possessed, during most of the sun's history, a lifetime (measured until further synthesis) not much less than that of the sun itself; otherwise some heavier element would be more abundant. Thus if we suppose the sun not to have changed appreciably in its more recent history, the lifetime of oxygen must be about 10^{10} years in the sun now, if not more. Since, however, the elements immediately preceding and following are nearly as abundant, some synthesis is probably occurring in all this range, and the lifetimes cannot be appreciably more than that of the sun in any instance.

The amount of any element converted per second to the next higher is equal to the quotient of the total amount of it present and its average life. From, say, *Li* to *N* this would be roughly a constant, if conditions had remained the same long enough; but however long they have been the same, it cannot have been long enough for the most abundant element of all, unless the next one is just in the process of outstripping it, which we shall find is unlikely. From about

[1] No great stress can be laid on the exact coincidence, although Gamow's figure (*op. cit.*, p. 639) does put Zn^{60} as the first unstable element. In point of fact we shall see (§5, C, iv) that Fe^{52} and Ni^{56} (which would certainly be "ionized") may also be unstable.

oxygen, or at any rate fluorine, we have assumed that the average lives start decreasing again; since in addition each element from nitrogen onward is to retain many or most of the atoms that reach it by synthesis, the amounts present immediately above $Z=7$ must fall farther below the equilibrium amounts the larger Z becomes. In fact, if we draw in Figure 2 a curve representing the equilibrium amounts, i.e., with ordinates given by the (logarithms of the) assumed average lives, and below it an idealized (smoothed) curve representing the actual amounts present, the difference (taken negative) between the ordinates of the two curves at any element Z is the logarithm of the fraction M_z/M_7, where M_z is the number of atoms of Z transformed to the next higher element per second, and M_7 is the corresponding number for $Z \gtreqless 7$.

It is apparent from Figure 2 that (neglecting some individual cases, especially *Ca*, which we discuss below) a simple increase of probability for process B with rising Z is consistent with the view that this fraction does in fact get steadily smaller from *N* to *Ti*, and then returns to unity. At each stage in this range then, more atoms are arriving than are leaving, i.e., the amounts present are increasing. In the iron group the elements are present in equilibrium proportions, but are increasing at only about one-hundredth of the rate at oxygen, since M_z/M_7 is small at scandium. In principle it is difficult to see how any synthesis theory in which the helium production occurred at the end of a long train could lead to stars that were stable for really extensive periods unless qualitatively exactly this state of affairs could arise; anything else appears almost bound to lead to a flooding of the helium supply.

ii) *Control of the supply of helium: expanding stars.*—So long as a star still has a fair proportion of its original hydrogen left, it will wish to keep practically constant the amount of that element which is producing helium. This means not merely that the net amount of Zn^{60} should not be increasing, but that not even the amount of Ni^{56} should be increasing although an atom of this element is produced for every α-particle emitted. Thus the elements immediately below this point must be present in such small amounts that in spite of their fairly short average lives they do not seriously add to the helium-producing cycle at all. On the other hand, the average

lives and amounts of Ni^{56}, Fe^{56}, Co^{57}, Ni^{58}, and Cu^{59} must be such that on the average a synthesis from each one to the next occurs for every *He-Li* synthesis, and thus also for every *B–C* or *C–N* synthesis; that is to say, the amounts of Ni^{56}–Cu^{59} must be approximately in equilibrium with the amounts of the elements up to nitrogen; there must, therefore, be a group of elements below *Ni* that are present in considerably less than equilibrium proportions.

It may however still be asked whether a star will in fact be able to arrange matters so that this situation develops. A little consideration shows that it will, but the somewhat surprising conclusion also emerges that probably it spends a large proportion of its life very slowly expanding, not contracting.

Let us picture a star which has just settled down into its steady state after the large initial expansion. Most of its mass is still hydrogen, but there is enough of each of the elements up to *Zn* for the *Fe–Zn*^{60} syntheses to occur at a suitable rate; the supply of *He* is adjusted to the star's energy needs. There will however then be a synthesis of Fe^{56} from below, in addition to the re-formation from Ni^{56}, and this will increase the helium-production a little. The star will counter by expanding slightly, which decreases the rate of all synthesis processes, and thus does three things at once: (*a*) it retards (temporarily) the development of energy, for a given rate of production of helium, by slowing down all syntheses; (*b*) it retards in particular the *Fe–Zn*^{60} syntheses, and hence the rate of production of helium; (*c*) it checks the formation of Fe^{56} or Ni^{56} from below, by interposing a "wall" in Figure 2, over which it is increasingly difficult for elements to climb; this wall must clearly come where synthesis is least probable, i.e., where process B is just beginning to outweigh process A.

Not until this third object, the erection of a "wall" in the oxygen region, has been attained will the star have the situation really in hand; it is essential to reduce to a very small value the rate of (original) formation of elements producing helium, and this can only be done by holding up the synthesis at some intermediate elements, namely, those from *O* to *Mg*, almost indefinitely. But for a given average life of any of these atoms the number that are built up past this point is proportional to the number present, and is therefore

rising steadily; this then must also be steadily countered by expansion. If the expansion is overdone, the synthesis of Zn^{60} will also be unduly retarded, the supply of *He* will fall, and the star will have to contract to obtain its energy; if the expansion is inadequate, too much new Fe^{56}, and thus too much Zn^{60}, will be formed, and the increased *He* supply and energy generation will have to be countered by further expansion. Over periods long compared with the natural pulsation period of the star, the situation thus appears to be intrinsically stable.

We will now consider also the question of exhaustion of the hydrogen. If we transfer to atom nuclei the language of chemistry, we may say that stellar energy is provided by burning elements in an atmosphere of hydrogen (the products of combustion being slightly heavier elements), and the law of mass action is just as simply applicable here as in more familiar cases. Other things being equal, a constant rate of *He* production will follow if the product of the partial pressures of *H* and, say, Fe^{56} (or Cu^{59}, etc.) is constant; if the individual synthesis probabilities are in fact decreasing, it is this product which must rise to correspond. Obviously, the product is at first very much more sensitive to changes in the absolute amount of the heavier elements than to changes in the amount of hydrogen, and very small increases in the absolute amount of iron will be ample to compensate the corresponding loss of *H* and so keep the product constant. Thus the increases in the *Fe–Zn* amounts will at first be due almost entirely to the need to compensate the slow decrease in synthesis probability, which, in its turn, is caused by the expansion demanded by the rising amounts of *N*, *O*, *F*, etc. It is only when the amount of hydrogen has fallen to less than, say, half the star's total mass that it begins to have a serious effect on the product, and to demand an additional increase in the amounts of *Fe–Zn*. Ultimately, of course, the product can only be maintained at the necessary figure if the star contracts again.

iii) *Calcium and scandium.*—Although we have deliberately avoided a detailed treatment of each element, it is perhaps desirable at this point to touch on the amounts of *Ca* and *Sc*, since both are quite well established and the difference between them is very striking. In a series of elements present in equilibrium amounts, the

amount of each element is of course determined solely by the rate of synthesis from it to the next, and is quite independent of the lifetimes of previous elements. In particular, an element will not be scarce merely because its formation from the previous one is difficult; the previous element will simply build up to a suitable excess. Where, however, we have not equilibrium, but amounts increasing with time, this is not so; if the *Ca–Sc* transition is exceptionally difficult, *Ca* may be exceptionally abundant and *Sc* exceptionally scarce. Since we have from the first admitted the possibility that in some cases the chance of a proton's remaining in a nucleus when it gets in may be in fact comparatively small, and since the one assumption that this is so for the *Ca–Sc* step covers two different phenomena, we may perhaps introduce it; the equilibrium curve in Figure 2 should then have a decided peak at *Ca*, resuming its normal course at *Sc* (no minimum!), and the peak should be higher above the actual amount at *Ca* than the separation of the two curves at any previous element. The consequent drop at *Sc* is then an additional guaranty that the helium-producing group of elements is still living in a fairly complete state of "splendid isolation." It would in fact seem natural to expect that when these elements finally have to be more rapidly added to, the scandium gap should first have to be filled up; an increased amount of *Sc* would then, unless there are further complications, indicate an aging star.[1]

iv) *The iron group.*—The fact that the maximum in the iron region is somewhat wider than this simple theory demands is no difficulty. In order to obtain radioactive elements at moderate Z-values it is necessary that the probability of electron penetration should be appreciably less, on the average, than that of proton penetration, but it need not for that reason be vanishingly small. Also it is doing no violence to Gamow's theory if we suppose that the first unstable element is one, or even two, places earlier than his estimate. Thus Fe^{52}, Ni^{56}, and Zn^{60} may all be unstable,[2] and the synthesis process

[1] It might of course also be a sign of an exceptionally young one.

[2] Their absence on the earth does not guarantee this. They may have taken up electrons, and this does not come under the head of "instability" as we have been using the word.

may sometimes arrive at one and sometimes another, according as electrons do or do not succeed in penetrating the nucleus half as often as protons. The disintegration of Fe^{52}, followed (if it were) by the incorporation of two electrons, would result in Ti^{48}, but sometimes Ni^{56} or Zn^{60} will be reached instead of Fe^{52}; on the whole, a steady rise in abundance from titanium until, perhaps, iron is thoroughly to be expected. There is however no very obvious explanation even so for a marked peak at iron.

D. NOTE ON THE INITIAL CONTRACTION

We have assumed in this investigation that the contraction in a star's very early history would not be extreme; if it were, it would both occupy a considerable part of its lifetime and also require a large amount of synthesis afterward to retrace. It is easily shown that on our assumptions the present situation is consistent with the view that the sun did not suffer a very excessive contraction.

The contraction will stop fairly soon after some elements (presumably those from *Pb* to *U*) have begun to supply as much *He* as the sun needs for synthesis. Under present conditions the elements in the iron region are abundant enough to do this; if the heaviest elements (whose total amount is much the same now as it was at maximum compression) were at that time abundant enough, the synthesis-times must have been say one thousand times shorter for them then than now, if the probability of process B takes roughly the course shown in Figure 2. An increase of this magnitude would be obtained by a rise in temperature of less than 50 per cent if the probability varies even with only the twentieth power of the temperature; it must presumably vary also with the first power of the density (if not more), but the central temperature will probably have been influenced by the smaller mean atomic weight at the time, which will perhaps compensate this advantage. Roughly, then, the sun's minimum radius will have been two-thirds of its present one, the time spent in contracting to that point will have been less than 10^8 years (even allowing for the delaying effect of some synthesis in the latter part of it), and the total amount of synthesis necessary to recover from it will represent well under 1 per cent of the present amounts of oxygen and the light metals. The position in this respect thus appears quite satisfactory.

It may be noted that since Figure 2 is plotted on a logarithmic scale, 90 per cent of the sun's life has been spent in adding the top unit to the various ordinates, so that the actual appearance of the curve is much as it was at the time when the earth is believed to have left the sun.

The possibility that the expansion which must follow this contraction should be very violent suggests that the appearance of the star during it could be that of a nova. We do not, however, consider that the novae that have been observed can in fact usually be explained in this way. For, on the one hand, they seem to be too numerous; the pre-nova stars should then also be numerous, should show (probably) nothing but hydrogen in their spectra, and should avoid the usual regions in the Russell-Hertzsprung diagram. And, on the other hand, novae seem usually to have been dwarfs before the outbreak, whereas we shall find the most violent expansion is to be expected at large masses. It appears preferable to explain the phenomenon as being due to a sudden generation of energy not very deep in the star, consequent on a fall of meteoric matter into it (cf. § 7, *C*).

If we abandon the assumption of process B, the contraction necessary to form uranium by process A is so large that an entirely prohibitive amount of synthesis would be necessary to recover from it. Also, the sun would in that case have no helium supply still in action now.

We have now apparently accounted (in the case of the sun) for the amounts of all the elements except helium; the discussion of this must be postponed to a later section. But in point of fact neon also appears to offer some difficulty, for it ought on our theory to be one of the commonest elements in the sun. It is true that we could not expect to observe it readily (though even in the sun it might well show up as easily as helium); but there are hotter stars where we might expect it, and it has not been found there. Possibly it is in somewhat the same position as scandium; it cannot however be denied that there may be a serious difficulty here (cf. also § 8, *D*, i).

6. STELLAR TYPES AND MAGNITUDES

A. SURVEY OF DATA

We turn now to the other side of the question, namely, the problem of accounting for the various types of stars. In order to get an

orientation for the discussion it is desirable to redraw the familiar Russell-Hertzsprung diagram on a slightly more physical basis than was originally possible. Even now, lack of data makes this difficult, but enough can be done for our purpose.

As independent variable we choose the mass of the star, since on our theory this is obviously much the most fundamental and unalterable of the various stellar characteristics. As dependent variable we want something which will give us as much new information as possible, so that certainly we shall not choose the luminosity. Spectral type, or surface temperature, would also be unsatisfactory as being comparatively secondary phenomena. Central temperature would be ideal if we knew it, but here the theoretical uncertainty is too great. There remain radius and mean density, between which there is little to choose. On a logarithmic scale both the lines of constant radius and the lines of constant mean density are straight, parallel, and equidistant, and we may make either of them perpendicular to the lines of constant mass as we please. It is perhaps slightly more convenient to choose the density lines.

In order to obtain a sufficient number of points we have to rely on the mass-luminosity relation, which of course introduces some uncertainty. More serious, however, are the uncertainties in the actual effective temperatures to be assigned to the various spectral types (especially since different observers differ in the spectral type to be assigned to even a given star), the correction to the M–L relation depending on spectral type, and perhaps the correction from visual to bolometric magnitude. It is also becoming clear that, for a given type, the dependence of surface temperature on the mass is not adequately taken care of merely by dividing the stars into giants and dwarfs.[1] As a result of all these factors our diagram is necessarily somewhat uncertain still, but roughly we get the result shown in Figure 3, where the shaded areas represent types of star that are common.

If all stars had the same mean molecular weight, and if the M–L relation held strictly, the figure would be a distorted reproduction of the familiar one, with a one-to-one correspondence of points. This is not quite the case, though it holds roughly. (The usual

[1] Sticker, *Zeitschrift für Physik*, **61**, 557, 1930.

orientation is obtained for the main sequence by turning the figure through about one and a half right angles counterclockwise.) The correspondence is close enough to permit the principal spectral classes and the magnitudes to be sketched in roughly, but not close enough to prevent considerable overlapping of adjacent types and magnitudes in individual cases.

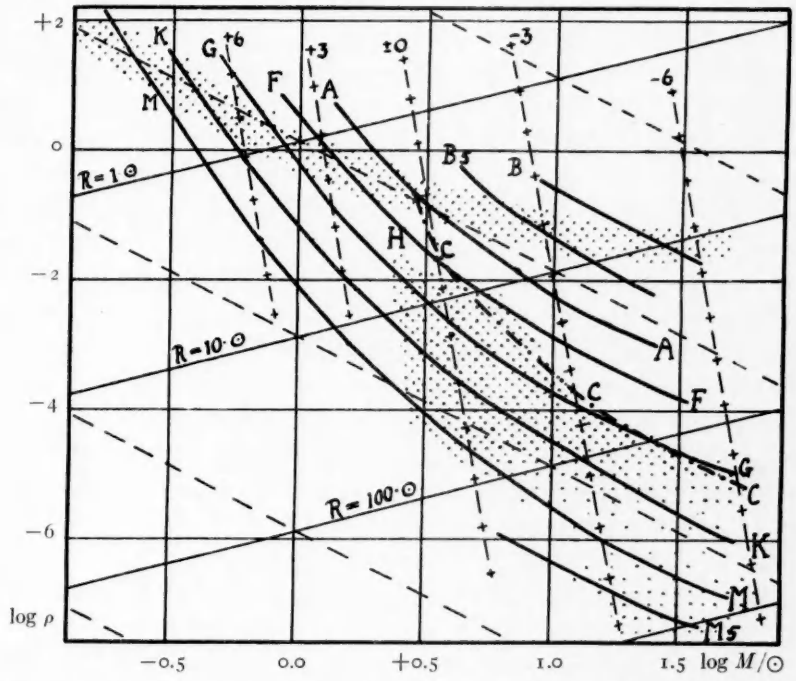

FIG. 3.—Stellar types and magnitudes. The dots indicate the supposed distribution if all stellar masses were equally common. *C* denotes the Cepheid line, *H* the Hertzsprung gap.

The lines of constant radius are shown in the figure, and points can of course be referred to this oblique system of co-ordinates if desired. If all stars had the same mean molecular weight, and were also homologous, it would be possible to draw the lines of constant central temperature as well, simply from Lane's laws[1] and without any regard to the validity of the M–L relation. In the absence of radiation pressure they would also be straight, parallel, equidistant,

[1] See, e.g., Russell, Dugan, and Stewart, *Astronomy*, p. 883.

spaced as far apart as the lines of constant radius, and sloping downward twice as steeply as these slope upward. Lines of this type have been included in the figure, though it is not at all suggested that they really represent lines of constant temperature. At any one mass, their spacing does however probably give a fair temperature scale. Radiation pressure will cause the true lines to bend somewhat upward toward the right-hand end, so that they follow nearly the course of the main sequence if the model is the same throughout; probably the effect is not enough to involve a quite constant central temperature along the whole main sequence including the B and O stars, so that the central temperatures do rise a little with increasing mass, but not very much. If the model changes systematically with mass the constant-temperature lines will of course be correspondingly distorted, probably in the opposite direction.

It is at once apparent that (as is of course a well-known consequence of Eddington's theory) the stars fall into two classes. These are not however the giants and dwarfs of current phraseology, since the former usually include all bright stars; the distinction is between those whose central temperature is a well-defined function of their mass (and perhaps nearly constant for all masses) and those whose central temperature is ill defined but appreciably lower. Moreover, there is a gap between the two groups at all masses at which both occur; this is in part the "Hertzsprung gap," but the empty region above the cepheid line now appears as a sort of continuation of that. This region was of course almost bare of stars in the original diagram too, but its shape made it less conspicuous there; it figures also in a theory given by Jeans.[1]

B. THE MAIN SEQUENCE

i) *Lifetimes and helium formation.*—We will consider first the main sequence, i.e., all those stars, of whatever mass, whose central temperature, while its absolute value is unknown, is, according to Eddington, approximately that of the sun. We naturally attempt to show that the process of energy generation can be the same for all of these stars as for the sun.

At first sight the principal obstacle to this view might seem to be

[1] *Monthly Notices of the Royal Astronomical Society*, **87**, 400, 1926.

the very considerable differences in the rate of energy generation per gram, as between M dwarfs and large B and O stars; these can however be taken care of, even though the change of density is in the wrong direction. For in addition to the rapid variation of all synthesis probabilities with temperature we are dealing in effect with bimolecular reactions; a change in the partial pressure of helium is an additional effective means of changing the rate of energy generation, and if the helium supply is adequate the correct concentration of helium will soon follow automatically. The main difficulty, just as it was with the sun itself, is not the immediate question of the generation of energy, but the more remote one of the formation of helium-producing elements.

This demands that whatever the age and the mean density of a (main-sequence) star, some element in the oxygen region must have a lifetime about as long as the star itself. It would be possible to assume that as a matter of fact no stars yet are older than 10^{10} years, and that small dwarfs whose hydrogen supply in itself would allow 10^{13} years of life may be left to deal with the situation when it arises; we shall not go to this extreme, but shall content ourselves with supposing that no stars are much older than 10^{11} years at present; even this age creates some difficulty, since the heaviest B stars cannot on our assumptions live longer than about $10^{8.3}$ years, so that on this account alone there will be a factor of nearly a thousand in the average life of oxygen in different stars.

In addition, however, we can hardly escape the assumption that synthesis probabilities must vary as the first power of the density at any rate. These two considerations involve that the average lives of the longest-lived elements in different main-sequence stars, reduced to constant density through the whole range, must differ by a factor of at least 10^5, and quite possibly 10^6; this has to be taken care of by the temperature alone.

Even the thirtieth root of a million is scarcely less than 1.6, and Figure 3 indicates that, if the stars are homologous all down the main sequence, the temperatures probably do not vary quite over a range of 1.6 to 1 even in the entire range from β Centauri to Barnard's star. (The effect of radiation pressure is of course considerable in the heavy stars, but the actual values for it cannot be

taken directly from Eddington's work, owing to the change in the assumed hydrogen content.) Since in addition we cannot explain the diffuse giants quantitatively at all unless the central temperatures rise more rapidly with the masses than they would for homologous models, we are led to inquire how much the model may actually change.

ii) *Models dependent on the mass.*—Jeans pointed out some years ago[1] that Eddington's well-known approximation $\eta k = \text{Const.}$, which led to the value 3 for the polytrope index n at all masses, ought probably to be modified to take account of Kramers' theory for the absorption coefficient k. The reason why Jeans's suggestion has not been more widely adopted than it has as yet is probably that Kramers' theory and the absolute values for k in stars seemed to be in serious disagreement; but Russell's discovery that the proportion of hydrogen in stellar matter is much higher than had been supposed seems to remove this difficulty entirely. We shall therefore follow Jeans's theory in this respect, though not in detail; he took for simplicity the case that η, the average energy generation per gram inside a sphere of radius r in the star, was constant, or nearly so, whatever the value of r, and we shall find that the source is probably in fact appreciably too much concentrated to the center to allow this.

The detailed investigation of this matter must be left to a later paper. Qualitatively, however, it is clear that the position is fairly closely what our theory requires. Jeans found that the polytrope index should vary from $3\frac{1}{4}$ for very small stars to 7 for very massive ones, and his formulae show that if the energy source is concentrated toward the center, these figures will be lowered. There is in fact some reason for supposing that even for the sun n is less than 3, and of course it cannot actually be greater than 5 in any star.

The central temperature for polytropes of mass M and radius R is given by

$$T = \frac{K\mu\beta M}{R}\, 10^{-16}, \tag{5}$$

where μ is the molecular weight and β the ratio of gas pressure to total pressure (nearly unity for all small M-values). K is a constant

[1] *Monthly Notices of the Royal Astronomical Society*, **87**, 36, 1926.

depending on the index n; it has the values 4.335, 4.85, 5.665, 6.90, and ∞, for polytropes of index $n = 1.5$, 2, 2.5, 3, and 5, respectively, and may clearly be expected to vary over a range of 2:1 at least in the course of the main sequence. Both the decrease in μ at all masses and the probable increase of n above the value 3 at large masses act in the direction of making β more nearly 1 than on Eddington's theory, and the change in temperature between any two stars will thus be practically as large as the change in KM/R. Reference to Figure 3 shows that on this basis the temperature change down the main sequence is appreciably larger than the minimum we must demand, and provisionally we may therefore regard the position as quite satisfactory for all these stars. (If the change is very large, process B must be less sensitive to temperature than we have assumed.)

iii) *Width of the main sequence.*—Our theory leads at once to the view that if all stars of a given mass had the same constitution and molecular weight, the central temperatures and radii would be quite sharply defined functions of the mass; decreasing the radius will increase both density and central temperature, and on both accounts the synthesis-times will therefore decrease, so that only one radius will result in the right synthesis-time for the longest-lived element. We must accordingly ascribe the observed "width" of the main sequence partly to differences in mean molecular weight, as would probably be done on most theories. But in our case the actual working-out of this specific problem is complicated, and will not be attempted in detail here; we shall merely show roughly how the various factors appear to interact.

The mean molecular weight of the stellar material will clearly rise steadily as the hydrogen disappears, and this will in general mean a rising central temperature if the radius stays constant, or an expansion if the temperature is to be kept constant. On the other hand, we have already seen that even with constant molecular weight a star will in fact be compelled to reduce its central temperature slightly during the earlier and middle stages of its history, and to contract again during the later ones. Roughly, it seems likely that the effect of changing molecular weight will not begin to be very noticeable until the other reason for expansion has spent itself

and comparative shortage of hydrogen is demanding a contraction; thus the earlier stages will be characterized by an expansion, owing mainly to the rising amount of oxygen and the need to prevent this from running on down to iron, and the later stages will see a contest between contraction required by shortage of hydrogen (on account of the law of mass action) and expansion required by rising molecular weight. Ultimately the contraction must win, but there seems reason to expect that until the hydrogen supply has run fairly low the issue will remain in some doubt, so that stars will not on this account be forced out of the main-sequence band until they are quite old.

The question whether a suitable total rate of energy generation is still within the star's control, when it has already fixed the synthesis-time for one element, is not more difficult for any other star than for the sun. If a suitable rate of helium generation is provided, the amounts of the various light elements will settle down at suitable values fairly rapidly. The "suitable" rate is of course only reached by trial and error; but, just as in the case of the sun, expansion, to which the star resorts in order to counteract the effects of an excessive amount of helium, also diminishes the rate of production of helium and of helium sources; trial and error is thus a feasible method.

iv) *Constitution of stars of different masses.*—The question whether the relative amounts of the various elements will be the same all down the main sequence cannot be answered for the heavier elements. For the others, a qualitative similarity is obviously to be expected, with the following main points at which differences might be looked for.

The effect of a change of temperature on the relative average lives for different elements is known, with process A; a decrease of temperature increases the differences between successive elements. Thus the relative amounts of the lightest elements should be more nearly equal in the B and A stars, and more markedly unequal in the K and M stars, than in the sun. This is, however, somewhat dependent on the assumption that the stars are at the same relative ages (e.g., all with half their hydrogen gone), whereas they are perhaps more likely to be at roughly the same absolute ages. Also, the heaviest stars of all (B and O types) are probably at all times in a

state of comparatively rapid readjustment, since their total lifetimes are so short, and for all stars the assumption that mixing is really complete for short-lived elements is dangerous.

Since we know nothing about process B we cannot say whether the same element will be the most abundant in all stars, but it is at least perfectly possible. The same may be said of the *O–Fe* ratio; if the dependence of process B on Z is not very sensitive to temperature, this will be nearly the same in all stars, but we cannot be sure. In the present state of our knowledge, it is as probable as anything else. The *Ca–Sc* ratio might be expected to vary in different stars, but it is difficult to suggest any detailed connection between it and the age of the star except for very old stars.

The relative amount of hydrogen is an index of the star's age; but throughout most of a star's life the hydrogen that disappears goes to form elements for which Z is not greater than 30. Thus if of two stars of equal mass one has more hydrogen than the other, the other should have more of the elements from carbon to zinc, but not more of the heavy elements. A comparison between stars of different mass is however of little value in this respect; and if there is in fact a small steady escape of elements upward past the instability at Zn^{60} the test may be valueless, even for the same mass, except in the *Pt–Bi* region, and here it is difficult to carry it out. We should however certainly expect that in such a case the star with less hydrogen should have, if anything, actually less of these elements also.

If the stars are all of about the same absolute age, the relative amounts of hydrogen should of course increase with decreasing mass; but we cannot be sure that the smaller stars are not, on the average, older. The test would also be far from easy here; the hydrogen estimate is decidedly uncertain even for the sun, and the possibility that mixing should be incomplete is greater for hydrogen than, at any rate, any stable element above boron.

In all other respects there seems little reason to expect any marked change from star to star, and in many ways there is good reason to regard it as improbable. The general run from *Li* to *Ge*, and the maximum at $Z = 55$–60, should be common features of all main-sequence stars. While then definite predictions are not easy to make,

the conclusion already reached empirically, that in general the proportions of the different elements seem to be roughly constant throughout the galaxy,[1] is quite what might be expected on our theory, at least for the majority of the stars.

v) *Value for the sun's central temperature.*—Our theory allows us to make an estimate of the sun's central temperature, since the dependence of process A on temperature is known, and the longest-lived element (which must still be partly governed by this process) must have a lifetime comparable with that of the sun. In order to perform the calculation we must however first correct the error already referred to in the paper of Atkinson and Houtermans.

The original theory treated the bombarded nuclei as "targets" of radius known from probes with α-particles; the "radius" was then that distance at which the assumed potential-curve had its maximum. Since however we are dealing with slow protons, this length is very much less than the wave-length of the incident wave-train, and also very much less than the minimum approach distance of a proton on the classical theory. The assumed radii were about $10^{-12.4}$ cm for light elements, the wave-length of a proton of velocity $10^{7.5}$ is[2] ant. $(0.8-27-0.2+24-7.5)=10^{-9.9}$ cm, and the distance of closest approach to even a helium nucleus is as much as ant. $(0.6+1.3-20-0.2+24-15)=10^{-9.3}$ cm. Since all protons aimed at a target of this radius round the nucleus will be very greatly deflected, and since the ψ-function begins to rise rapidly at this distance (in central impacts), it would seem natural to use this in rough estimates of the target area; we have however one further condition to observe, namely, that the angular momentum must not exceed about $h/2\pi$. (The Gamow formula holds only when the azimuthal-

[1] See, e.g., Miss Payne, *Stellar Atmospheres* (Harvard).

[2] This notation is new; it is however extremely convenient for approximate work. We simply write ant (x) for "antilogarithm of x" or 10^x, in exact analogy with exp (x) for e^x. The actual form of the notation was adopted after consultation with Professor Russell; it is very cheap to print. Thus the second equation in the text, written in full, would be

$$\frac{4\times(4.77\cdot 10^{-10})^2}{1.66\cdot 10^{-24}\times(3.1\cdot 10^7)^2}=5\cdot 10^{-10}.$$

The second figure behind the decimal point in the logarithms can be estimated rapidly and correctly with very little practice by the method suggested in *Nature*, **124**, 94, 1929, but this is unnecessary in the present work. Logarithms have of course also the advantage of constant percentage error for all numbers.

quantum number is 0; when it it 1, the angular momentum is $\sqrt{2}h/2\pi$.)

The latter condition is slightly more stringent than that obtained from minimum approach distances, and may therefore be used throughout. In effect this involves including the target area as a variable in the integration of Atkinson and Houtermans' theory; the effect of its variability is slightly to decrease the dependence of λ on Z and T. The effect of its very much larger absolute value is of course to increase λ quite considerably for any given Z and T; since, however, the dependence on T remains extremely great, the temperatures calculated by the corrected method are only about 30 per cent less than those obtained from the old formula. It thus appears unprofitable to go into great detail, and we simply give the final result, obtained by the rough process of repeating the analysis of Atkinson and Houtermans with the variable value $r=h/(2\pi mv)$ in place of the constant radius σ.

Instead of

$$\lambda = N\pi\sigma^2\bar{v}\cdot 2\xi^3 e^{-3\xi^2}$$

we now obtain

$$\lambda = \frac{Nh^2}{\pi^{3/2}m^2v_0}\cdot \xi e^{-3\xi^2},$$

$$\approx 10^{-5.5}\frac{N}{v_0}\cdot \xi e^{-3\xi^2}, \qquad (3)$$

where ξ, N, and v_0 have the same significance as before. It is convenient also to re-write (2) in the form

$$T = 2.86\cdot 10^9(Z/\xi^3)^2. \qquad (4)$$

The conclusion that λ varies approximately[1] as T^{ξ^2} is unchanged; the values of ξ in which we shall be mainly interested are however somewhat higher than before, so that the actual dependence of λ on T is more pronounced even than it was.

We may now calculate the sun's central temperature. Approximately we may put the central density at 30, of which half may be assumed to be hydrogen; the value for N is then ant. $(1.5\text{–}0.3+24\text{–}0.2)=10^{25.0}$. If we may roughly estimate the temperature at

[1] More accurately, as $\rho T^{\xi^2-2/3}$.

20,000,000°, v_0 will be about $10^{7.8}$. The average life for process A alone, if the equilibrium curves run approximately as in Figure 2, might be one hundred times as long as when both processes are acting, but this figure is uncertain. It is probably more than counteracted by the fact that not nearly all the oxygen atoms in the sun will be at a temperature high enough for an appreciable amount of synthesis to be possible at all; possibly only about one-thousandth of the sun's mass will at any one time be at a high-enough temperature, and there will be ample time for mixing to keep the oxygen atoms circulating in and out of this "active volume." Thus we may consider that if the sun's age is $10^{10.5}$ years, the average life of an oxygen atom that remained at its center, if process A alone were acting, would be about $10^{9.5}$ years, so that $\lambda = 10^{-17.0}$/sec. Solving (3) gives us, then, $\xi = 4.75$, and putting $Z = 8$ in (4) leads to the value

$$T = 16{,}000{,}000^\circ .$$

Using the older formula, according to which the target area for this velocity would be about one hundred times smaller, the temperature obtained is 20,000,000°; if the new formula is wrong, it probably represents an overcorrection,[1] and it is thus clear that no very great effect on the temperature is to be expected from further refinements of theory.

We may then compare this value with that obtained by Eddington's entirely different method. The figure that Eddington actually gave was 39,500,000°, but that was on the assumption of a mean molecular weight of 2.11; the large proportion of hydrogen will reduce this to a value in the neighborhood of 0.82. In addition, the polytrope model with $n = 3$ may represent too much concentration of mass toward the center in the case of the sun.

Using (5), with $\mu = 0.82$, $\beta = 1$, and the values of K given on page 289, we obtain for the central temperature the values 10,000,000°, 11,000,000°, 12,500,000°, and 15,500,000° for the polytropic indices $n = 1.5$, 2, 2.5, and 3, respectively. Since it is not improbable that the amount of hydrogen should be somewhat less than at the surface, so that the molecular weight would be more than we have assumed, the agreement is inside the limits of uncertainty of the theory of

[1] Cf. p. 257, n. 1.

polytropes; in view of the remarkable insensitivity of the results obtained by our method to changes in the assumptions involved, it is perhaps fair to regard the agreement as not merely accidental. Incidentally, the investigation of §6, B, i, ii shows that, as far as the temperatures can at present be determined, a similar agreement exists for all stars throughout the entire main sequence.

vi) *Summary of results for main sequence.*—In accounting for the main sequence we have of course dealt with the great majority of all stars (unless, in fact, the white dwarfs are the more numerous). It appears that in this family stars of all masses may occur, and very probably of widely different ages (with corresponding differences of mean molecular weight), but that of course there is no correlation between mass and age beyond that possibly imposed by the existence of a maximum age differing with the mass. Nor is there any appreciable correlation between exhaustion of the source of energy and intensity of the radiation; we have not had to call in an arbitrary parameter of this sort in order to account for the facts. The family consists essentially of stars in which the longest-lived element in the synthesis chain has an average life comparable, at any time, with the past lifetime of the star; this involves a systematic increase of central temperature with mass such as may apparently follow from Jeans's modification of Eddington's theory. The temperature is not quite definite at any one mass; it varies with age and molecular weight in a complicated way, but not over a wide range. Its value is about 16,000,000° for the sun, which seems to be in entire agreement with revised estimates based on Eddington's theory. The constitution of all the stars in the family should be approximately that actually found in the sun, differing little from star to star except in so far as age reduces the amount of hydrogen; minor differences that might be expected are in no case such as should be easily detected. All these conclusions follow, with some approach to rigor in some cases, and with considerable plausibility in all others, from our assumptions, and this degree of success alone might justify a preliminary theory in including one quite arbitrary assumption among its premises. We can, however, give a fair account of the giants of low density also, along essentially the same lines.

[*To be continued*]

REVIEWS

La planète Mars. By E. M. ANTONIADI. Paris: Librairie Scientifique, Hermann & Co., 1930. Pp. 240. 4°. Figs. 150. Pl. 10. Fr. 80.

In this volume Mr. Antoniadi has gathered the results of his studies on Mars with the 33-inch refractor (objective by the brothers Henry) of the Meudon Observatory near Paris. The author, who has been following the planet assiduously for many years as director of the Martian Section of the British Astronomical Association, attempts to contribute to the explanation of the Martian topography and physiography by coordinating his own results with those of previous and contemporary investigators.

In the first part of the book we find generalities about the planet. Being of Hellenic origin, the author, who has contributed a number of studies on early Greek astronomy, is well versed in the terminology of the ancients concerning the planet. Without transition the next chapter takes up the question of the best observing conditions and of the most suitable instruments for studying the planet. The advantage of large apertures in bringing out the fine details of the areographic markings is strongly emphasized. The author is known for his definite stand as a non-canalist, and his views on the presence or rather on the absence of canals on Mars are encountered all through the book.

The elements of the orbit (equinox 1920? not mentioned) of the rotation and the resulting seasons are tabulated in a third chapter, which would have come more logically ahead of the second. In connection with the first observation of the Martian phases by Galileo the rather startling assertion is found on page 13 that the telescope was invented by the Greek Zacharias Janssen of Middelburg.

The following chapters of the first section deal sketchily with the appearance and color of the dark and bright markings on the planet, with their seasonal changes and their probable origin. The "illusion" of the canals is dwelt upon more at length. On page 29 is found a striking illustration, on the one hand, of the curiously artificial aspect of the region Elysium as represented by Schiaparelli, with many straight and double canals, and, on the other hand, the same region resolved by the larger instrument in a world of complicated but natural-looking structure, which

required all the skill of the author in drawing, to be put on paper. Like a great many of the illustrations in this volume, this one had been published previously in the *Bulletin of the French Astronomical Society.*

The main part of the book is the second section, which covers fully three-quarters of the volume. Here we find the topography of the planet described in five subsections, to each of which corresponds an elaborate map. On these the names of the various planetary details are marked, with use of the nomenclature introduced by Schiaparelli, with additions by Lowell and Cerulli, as well as a few classical or biblical names proposed by the author. The maps represent an integrated picture of all the features seen at times, so that they show a great deal more than what is visible at any one moment.

A list of eighty-five reference points is given, the positions of which have been weighted from the determinations by Schiaparelli and by the author. The details themselves have been filled in on the map with the help of all the drawings available and a considerable amount of photographic material. The largest part of the book consists of the long list of surface features of all kinds, which are analyzed individually; for each of these we find the history of the discovery and of their changes. Many are illustrated by detailed figures of which there are one hundred and fifty in the text. At the end of the volume we find an alphabetic Index of all the designations used and of the principal authors mentioned, and, last but not least, a set of twenty drawings which are among the finest ever made of Mars. There are five groups of four each relating to five oppositions between 1909 and 1929. Few observers have been as successful as Mr. Antoniadi in reproducing all the exquisitely fine structure and the delicately fluctuating shadings that the eye catches when the planet is seen under fine atmospheric conditions in a powerful telescope.

The author does not touch upon the newer lines of attack in the physical study of Mars, such as the measures of radiation (Coblentz, Lampland, Pettit), or the analysis of the light of the planet in various wave-lengths (Wright, Ross).

G. Van Biesbroeck

Applications of Interferometry. By W. Ewart Williams. London: Methuen & Co., 1930; New York: E. P. Dutton & Co. Pp. viii + 104. Figs. 43. $0.85.

This is one of a series of monographs which aim to give an up-to-date résumé of the developments in the subjects considered. The treatment is

brief but unusually clear. The advanced student will find the subject sufficiently well covered, and numerous references will facilitate the study of any branch. Research and industrial workers in other fields will do well to study this little book with care, as some application of interferometry may be advantageous in their own work. And one trained in optics will find that this volume contains several valuable suggestions, and that the treatment is in some instances superior to that in most texts. The theory and uses of the Rayleigh interferometer are particularly well treated. It is to be hoped that in future editions the author may devote more space to his own important instrument, the reflection echelon. The reference list omits the work of Peters and his co-workers at the National Bureau of Standards; and the most recent determinations of precise wave-lengths. On the whole, however, the subject is remarkably well covered for so brief a discussion, and the book should be in all libraries of physics.

K. B.

Astrophysik auf atomtheoretischer Grundlage. By S. ROSSELAND. "Struktur der Materie," Vol. 11. Berlin: Verlag von Julius Springer, 1931. 8vo. Pp. vi+252. RM. 19.80 (unbound); RM. 21.20 (bound).

This new book by Professor Rosseland is intended, primarily, to serve as an introduction to the problems of astrophysics for the physicist, but it will be of great value to the astronomer as well. It is a thoroughly up-to-date review of modern astrophysics as seen by a theoretician. The observational data are discussed briefly and only in so far as they are used in the theoretical discussion. The author has not attempted to cover all phases of astrophysics. Because of lack of space he has refrained from discussing the magnetism of the sun. We miss also a discussion of comets, which are, or at least promise to become, of considerable importance as phenomena of radiation by fluorescence.

The book is divided into six chapters. A short introduction of ten pages serves to acquaint the reader with the fundamental data of astrophysical observations. The spectral classification, the determination of effective temperatures, of absolute magnitudes, masses, densities, and linear dimensions form the subject matter of this chapter.

The second chapter, "The Physical Foundations of the Problem of Stellar Interiors," attempts, on the basis of modern physical theories, to predict the properties of matter under the conditions supposed to exist in

the interiors of the stars. Since very little is known concerning these conditions we must be satisfied with an approximate treatment. All that can be said with certainty is that the temperature inside the stars must be very high—much higher, in fact, than the effective temperatures of the outer layers which give rise to the observable continuous spectrum. It is of interest to find that many of the properties of matter can be predicted on the basis of this knowledge. Thus, it is possible to predict that most of the atoms will be highly ionized—stellar material must consist of atomic nuclei, stripped of the majority of their outer electrons, and of free electrons. It is even possible, as Eddington has shown, to predict the approximate atomic weight of the material, and this information is of considerable importance in theoretical developments. There are various other properties that can be predicted: the state of ionization of the material, the method by which it transports radiation, its viscosity and its absorption coefficient, are discussed to good advantage in this volume. The treatment of the absorption coefficient is of special importance: the author has attempted to elucidate the atomic (or electronic) processes concerned in the production of the opacity of gases in stellar interiors.

In the next chapter, "The Hydrodynamics of the Stars," the author applies the results of his preceding chapter to actual stars. He follows, in the main, the classical treatment of Eddington, supplemented by some of the newer developments. The recent work of Milne on the interior structure of the stars has not been included in the book; only a very brief note on pages 73–75 calls attention to the importance of considering the properties of the outer layers in connection with this subject.

The fourth chapter deals with the origin of stellar energies. To a physicist this part of the book should be especially interesting. Our attention is here called to the fact that stars radiate energy into space at an almost unbelievable rate. Thus every gram of the sun's substance produces, on the average, about two ergs per second, and this rate of radiation remains approximately constant over very long intervals of time. Geologists and paleontologists suggest that the earth has received a substantially constant amount of energy for some sixteen hundred million years. No wonder, then, that all ordinary sources of energy, such as contraction, radioactivity, etc., are completely insufficient to account for the maintenance of this rate of radiation over such long periods of time. Professor Rosseland has briefly discussed various sources of information concerning the age of the stellar universe, and has reviewed the hypotheses suggested by several investigators to explain the maintenance of stellar energies.

Nearly one-third of the book is devoted to the "Physics of the Stellar Atmosphere," a brilliant exposition of present-day theories dealing with the outer layers of the stars. The equations of radiative equilibrium are discussed at some length, and many important features of the theory are pointed out. The results are then applied to the formation of the continuous spectrum of a star, and especially to the processes producing stellar absorption lines. The theoretical contours of spectral lines are discussed, and the theory of ionization is brought into use, in order to explain the spectral classification and the differences between giants and dwarfs. The problem of the chromosphere is next taken up, and this is followed by a short discussion of emission lines in stellar spectra.

The last chapter, on "The Problem of the Gaseous Nebulae," gives a lucid account of the application of the theories of radiative equilibrium and ionization to gases not directly connected with the stars, but forced to radiate by fluorescence, or by other atomic processes. There is an account of Bowen's work on the spectra of the nebulae. The last section of this chapter deals with diffuse matter in interstellar space.

It should be especially noted that the author has in many places supplemented the established data by independent discussions of his own which have not been published elsewhere. These include useful suggestions which will be greatly appreciated by the practical worker. The book will be highly valued by all those who are interested in the development of theoretical astrophysics.

O. STRUVE